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ABSTRACT

This is volume two of a three-volume set for teachers using SMSG junior high school text materials. Each unit contains a commentary on the text, answers to all the exercises, a copy of the questionnaire used for evaluating the material, and a summary of comments by the teachers using the text. Unit topics include: (1) non-metric geometry; (2) informal geometry; and (3) measurement and approximation. (MP)

MATHEMATICS
STUDY GROUP

JUNIOR HIGH SCHOOL MATHEMATICS UNITS

VOLUME II, GEOMETRY

Commentary for Teachers

SMSG

JUNIOR HIGH SCHOOL MATHEMATICS UNITS

VOLUME II, GEOMETRY

Commentary for Teachers

Prepared by the SCHOOL MATHEMATICS STUDY GROUP
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1959

These units were prepared in the summer of 1958 at Yale University by a School Mathematics Study Group writing team. The units were taught in a number of classes during the academic year 1958-59. No major editing of them has been attempted, but typographical and other errors have been corrected.

When the units were used, the teachers were invited to submit comments concerning their experiences. A questionnaire was filled out by teachers for each unit taught. A copy of this questionnaire and a summary of comments by the teachers is included for each unit.

In addition, in those cases where unit tests were prepared during the year, a collection of suggested test items appears at the end of the commentary on the unit.

This volume includes the units concerned with geometry. These are units VI, VIII, IX, X in the numbering system originally used.

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UNIT VI

NON-METRIC GEOMETRY

Sets and the Intersection of Sets

Length, area, volume and size of angle involve ideas of measurement. In this unit an attempt has been made to lead the pupil to consider some important geometric properties before he studies metric properties of polygons, circles, and solids. This is done in the spirit of teaching in which there is an attempt to lead the pupil to the discovery of unifying concepts as a basis for learning some of the more specific details in computation, or in the study of geometric properties and applications of these to a variety of problem situations.

Study of this material should lead to better understanding of metric properties of geometry when they are introduced both in the junior high school and in the 10th grade. It has been traditional to teach these ideas when they are needed for a particular geometric discussion.

But too often the teacher has assumed these properties are obvious or clear without mentioning them. Also there should be some advantage in considering together this group of closely related and analogous properties and observing relations among them.

The study of some aspects of non-metric geometry has become a separate mathematical discipline known as projective geometry. This kind of geometry illustrates a property characteristic of much of mathematics in that it had its origin in a very practical problem but has been developed in a way that serves as a basis of some very abstract mathematical theory. As is noted in the text material the origin of projective geometry is traced to some of the thinking artists of the Renaissance period about

perspective drawing. Many elementary but interesting results are obtained through a systematic study of projective geometry. For example in projective geometry points of curves which can be obtained as sections of a cone (parabolas, ellipses, and hyperbolas) can be constructed by use of straight lines only. A system of coordinates can be introduced without the use of a scale or any measurement whatsoever.

The teacher may find it helpful to read one of the following:

Courant, Richard, and Herbert Robbins. WHAT IS MATHEMATICS? Oxford University Press, 1941. Chapter IV.

Young, John Wesley. PROJECTIVE GEOMETRY. The Mathematical Association of America, 1930. Chapters I, II, and III.

CONCEPTS

Sets and the Intersection of Sets

1. We use line to mean straight line and we think of a line as unlimited in extent.

2. ~~A set contains elements which are collected according to some common~~ property. A line is said to be a set of points. The elements of this set are points. A set may have no element and be called an empty set.

3. The common elements in two or more sets make up the elements of the intersection set of the two or more sets. The intersection set may be the empty set.

Line Segments and Half-Lines.

4. A line segment is determined by two points on the line, called the endpoints of the segment. We use the idea of line segment to represent such things as a mark on the blackboard drawn along a straight edge or the edge of a book or of a desk.

5. There is always a third point between two points on a line.

6. A point on a line separates the line into three sets: the point, and two half-lines determined by the point. If the point is joined to the set of points on a half-line we have a ray. The point is called the endpoint of the ray. The intersection set of these two rays on the line is the point which is their common endpoint. Two half-lines determined by one point on a line intersect in the empty set.

Lines on a Point

7. Just as we say "a point lies on a line," we also say "a line lies on a point." In a plane a set of lines lies on a point. A plane is our ideal way of thinking about a flat surface like a table top. A plane is unlimited in extent.

8. There is a one to one correspondence between the lines on a point and the points on a line.

Closed Curves

9. A straight line is a curve. Broken lines such as those which we see in statistical graphs, triangles, rectangles, circles and figure eights are also curves.

10. Circles, triangles and figures like  are simple closed curves. A simple closed curve divides the set of points in a plane into 3 sets: the points inside the curve, the points outside the curve, and the curve itself. The curve is called the boundary of the inside and the outside. If a point, A, is outside a closed curve and a point, B, is inside the curve, then the intersection set of AB and the curve contains at least one element.

11. Closed curves which are not simple closed curves appear to cross over each other, like a figure eight.

Planes on a Line

12. On a line is a set of planes. If the intersection set of two planes is not the empty set, it is a line. We say a line lies on a plane, or that a plane lies on a line.

13. Two lines which do not have the same direction and for which the intersection set is the empty set are called skew lines. They must be in different planes.

Special Figures

14. A complete quadrilateral consists of four lines and six points on which the lines lie so that on each line are three of the six points and on each point are two of the four lines. A figure consisting of 10 lines and 10 points such that on each line lie 3 of the 10 points and on each point lie 3 of the 10 lines is called

the Desargues configuration. In the figure each of the 10 points plays exactly the same role. This property is also true of each of the 10 lines.

SKILLS

1. Use the word line correctly.
2. Be able to think about the points on a line as a set of points. Know the use of the term, "empty set".
3. Be able to find the intersection set of two or more sets of points on a line.
4. Recognize line segments and their endpoints.
5. Be able to identify half-lines, rays, and endpoints of rays and to determine the intersection sets of two or more of these sets. Know what a triangle is. Know what an angle is.
6. Use the word plane correctly. Be able to talk about a set of lines on a point.
7. Be able to identify a one to one correspondence, particularly among points and lines.
8. Use the word curve correctly.
9. Be able to identify simple closed curves and to talk about the inside and the outside of simple closed curves in a plane.
10. Be able to identify closed curves which are not simple closed curves.
11. Recognize that on a line is a set of planes. Be able to identify the intersection set of two or more planes.
12. Recognize examples of skew lines.
13. Be able to follow directions in drawing figures not involving metric concepts.

ACTIVITIES

1. Give illustrations of lines, and points on a line. Draw a line on the blackboard and mark points on it. With each example talk about how we think about a line as unlimited in extent. Emphasize frequently that we use the word, line, to mean straight line.
2. Talk about sets of a variety of kinds such as the points on a line, the pupils in Roosevelt High School, the members of Congress, kinds of fruit, counties in a state, television programs, and numbers.
3. Ask the class to find the intersection set of two or more sets. You might use examples like the following;
   (a) The elements which are common to the set of natural numbers between 19 and 50 and the set of natural numbers which are divisible by 7.
   (b) The girls in the room and the pupils with brown eyes.
   (c) The three sets: The cities of population over 20,000 in your state.
   The cities in your state with first letter of their names before J in the alphabet.
   The cities in your state which are south of the city in which you live.

   Choose some examples in which the intersection set is the empty set.
4. Have examples given of things which represent line segments and for each identify the endpoints of the segments. In a number of the examples suggest that the pupils imagine a line segment being extended to form a line.
5. For some of the examples of line segments ask the class to identify points on the segment between the endpoints. An example might be

a tightly stretched telephone cable, with the points being where the cable is fastened to two poles, or between any two poles there might be a bird sitting on the cable. Ask a number of questions about the numbers which are between two rational numbers. Try to emphasize that the concept of points on a line is quite similar to the concept of an infinite set of numbers.

6. Draw on the blackboard a number of lines, mark points on them and point out half-lines, rays, and endpoints. Ask why the term half-line can be used. We are of course safe in using this term since no one can ever measure or compare two half-lines to see if their lengths are equal. Have children come to the blackboard to illustrate these terms. Give examples and ask the children to give others of the use of the term "ray" in life. In each case identify the endpoint of the ray. Ask questions during all of the discussion about the intersection sets of two rays, two half-lines, or of a ray and a half-line. The intersection set of a ray and the line on which it lies is of course the ray itself. Include in your questioning examples of this kind. Discuss some of the exercises in class.

7. One of the purposes of this unit is to develop use of vocabulary and to encourage the disposition and ability to make simple, direct statements of mathematical properties. Pupils should be strongly encouraged to use expressions like "a point lies on a line" and "a line lies on a point." The language of sets should also be used whenever possible. Considerations up to this point have been in one dimension. One can use the example of a bug crawling on a wire or something of this kind for the line, and then change to life examples in which movement is restricted to a flat surface,

two dimensions. If possible make use of the soap film and a wire frame since this more nearly approaches the mathematician's idea of a plane.

8. If pupils have not had experience with one to one correspondence with numbers it will be well to talk about examples with numbers before making reference to the sets on a point and on a line. Some students may bring up the notion that the line of the set on A which is parallel to the line on which a correspondence with points is to be established, will not intersect the line and hence there will be one exception in the one to one correspondence. If this comes up the teacher might mention that in projective geometry where these correspondences play an important part, one assumes that parallel lines meet in an "Ideal" point and only one such point, in order to maintain the one to one correspondence. Parallel lines are not mentioned in the text material since this involves a metric property. If "parallel" is brought into the discussion by the student, the special cases might well be explored to some extent and it should be pointed out that these cases will be treated in detail in a later unit. There would be no harm if the class wished to assume that parallel lines meet in an ideal point.

9. Emphasize our broad use of the word "curve" and that a straight line is one kind of curve.

11. As geometry has been traditionally taught concepts of closed curves and the inside and outside of a curve have not had a very important role. However these are very useful notions when the set language is used in discussing triangles, area, and angles for example. Distinctions between simple closed curves and closed curves should be made only by example with no attempt made to point out some of the

difficulties inherent in curves like the figure eight. The teacher should realize a circle is actually a metric concept. However, since a circle is a simple closed curve so useful in examples it has been brought into consideration here. Any other simple closed curves like,  , would of course be just as useful for our purposes but we don't have familiar names for them. The notion that "if A is outside a simple closed curve and B is inside, the intersection set of A B and the curve contains at least one element", is an important idea and many questions should be asked about this relationship. The concept that a line which does not lie on two vertices has at most two points of intersection with a triangle, as we define a triangle, will come up in later considerations of geometry and hence should be made entirely clear.

12. Work with planes and lines in space is often more difficult than the other considerations up to this point. Models will be very helpful to many children. Pupils should be encouraged to make their own models. The teacher can use his own judgment about encouraging the drawing of figures representing intersecting planes. This may be helpful to some. Emphasize the analogy between lines on a point and planes on a line, also that the intersection set of two lines is a point and that the intersection set of two planes is a line. Seek opportunities to make analogous or "parallel" statements like this.

13. The question of parallel lines will probably come up again when skew lines are discussed. Parallel lines are not skew lines. One

distinction which might be pointed out is that two parallel lines like two intersecting lines lie in a common plane, or determine a plane. Two skew lines do not lie in a common plane and do not determine a plane. You should point out that some of the edges of a chalk box, for example, are skew lines in pairs, and others are parallel in pairs (or groups of 4). A complete discussion of this situation should be helpful.

14. You probably will not want to teach the section on Special Figures to all pupils. Pupils may not be able to see the significance of the complete quadrilateral and the Desargues configuration but they should enjoy working with these figures, and if they do, they will gain valuable experience with the "on" language and may come to see the beauty of figures in which no point or line has a special role. It may be interesting to the brightest pupils to observe that the complete quadrilateral is the section of a complete 4-plane in space by a fifth plane, and that the Desargues configuration is the section of a complete 5-plane in space by a sixth plane. A complete 4-plane in space consists of 4 planes (no 3 on the same line and not all on the same point) and the 6 lines of intersection of the planes in pairs. The intersection set of a fifth plane and each of the 4 planes is a line (hence the 4 lines of the quadrilateral) and the intersection set of the fifth plane and each of 6 lines is a point (hence the 6 points of the quadrilateral).

While the complete quadrilateral and the Desargues configuration seem like very different figures one is in a sense an extension of the other and both are among the most general figures of all of mathematics.

- Answers -

Exercises - pp. 2-3

1) a) 18, 19, 20, 21, 22

   b) --

   c) --

   d) the empty set

   e) square, rectangle, trapezoid, parallelogram, rhombus

2) a) 3, 5, 7, (or any other odd numbers)

   b) 0, 5, 10, 15, or any other multiple of 5 (including 0).

   c) Q, R, U (The student could label some points of Q, R, S, etc. his own and these would be correct)

3) a) 9, ~~10,~~ 11, 12

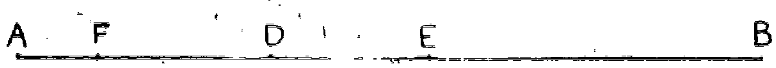
   b) For example, if telephone number is 5-0724, answer is 5, 7

   c) --

   d) P

Exercises - pp. 6-7

1) The ends of the street, possibly the intersection of Main Street with the city boundary, if Main Street goes through the town.

2) 

   c) yes

   d) no

   f)

Line segments	Point on segments
AB	A, F, D, E, B
AE	A, F, D, E
AD	A, F, D,
AF	A, F
FD	F, D
FE	F, D, E
FB	F, D, E, B
DE	D, E
DB	D, E, B
EB	E, B

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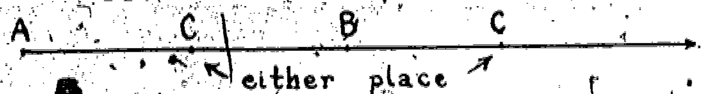
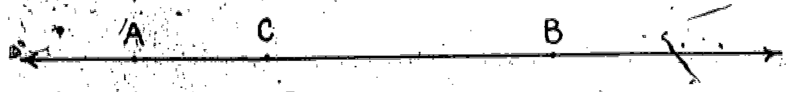
Exercises - pp. 6-7 cont.

3) 

c) It would be if we chose the C on the right.

4) 

a) The two half-lines determined by point A and the two half-lines lines determined by the point B.

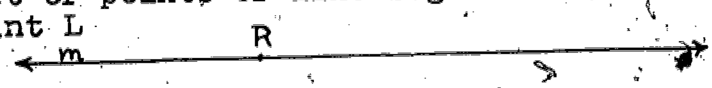
b) The set of points determined by:
  1) the point A
  2) the point B
  3) line segment AB
  4) the half-line from A through B
  5) the half-line from B through A
  6) the half-line from A opposite 4) above
  7) the half-line from B opposite 5) above
  8) and 9) the two rays determined by A
  10) and 11) the two rays determined by B
  12) the intersection of 4) and 5) (set of points between A and B exclusive of A and B)
  13) the line on A and B
  14) the points A and B

d) the half-line from A on B and the half-line from B on A

5) We might think of a ray of light as a straight line beginning at a source (the Sun) and extending without limit.

6) The ray could be described as the boundary line starting where it intersects with the Mississippi (inclusive) and extending eastward without limit, or at one of the end points of the line segment.
The half-line would be the same as the description above, except it would not include the intersection of the boundary line with the Mississippi River. One could also describe a ray and a half-line starting at the shore of Lake Michigan.

7) a) K, L, M
  b) L, M
  c) The set of points of line segment KM or LM and the ray with endpoint M
  d) The set of points of line segment KM or LM and the ray with endpoint L

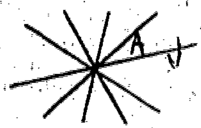


8) a) R
  b) The empty set
  c) The intersection set of the half line from R to the right and the ray determined by R (to the right of R) is the half line from R to the right.
  The intersection set of the half line from R to the right and the ray determined by R (to the left of R) is the empty set.

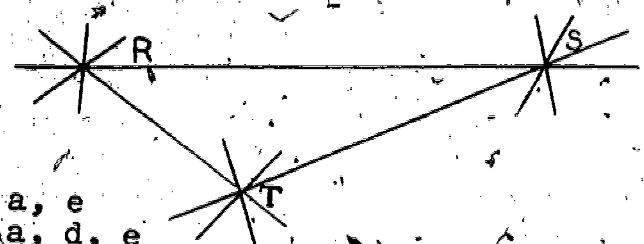
Exercises - pp. 9-12

1) intersecting streets, clock hands, airplane propellers, etc.

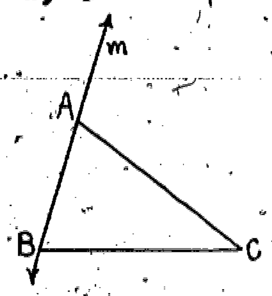
2) a sheet of paper, table top, walls, ceilings, floors, blackboard, etc.

3) 

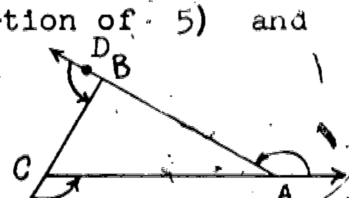
4)
a) unlimited number
b) unlimited number
c) --
d) set; four; elements
e) the line on A and B
f) intersection set

5) 

6)
a) a, e
b) a, d, e

7)
a) A
b) no
c) 

intersection set of $\overleftrightarrow{AB}$ and m is AB
intersection set of m and Δ is AB

8)
a) 6 (from A through B, from B, etc)
b) 6
c) the sets of points determined by
   1) the ray with endpoint A
   2) the ray with endpoint B
   3) the half line from A
   4) the half line from B
   5) line segment AB
   6) intersection of 5) and 2) (point B)
   7) point A
d) 

D on ray from B

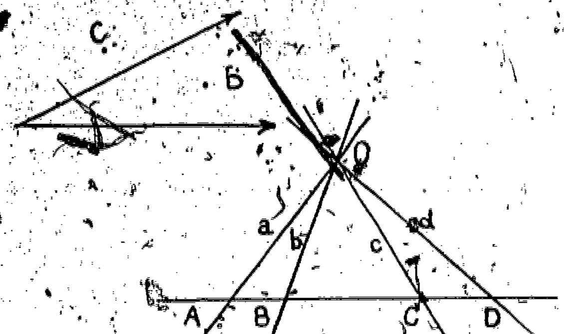
9) A quadrilateral is a figure composed of four points, no three on a single straight line, and four line segments joining them in pairs, such that no three line segments are on one point.

10) yes; if the three points are not collinear.

12) See diagram above (8.d)

Exercises - pp. 9-12 cont.

13)



14)

pt A corresponds to line a
" B " " " b
" C " " " c
" D " " " d, etc.

15) yes, <u>if</u> all the desks are filled, and there is one pupil for every desk and one desk for every pupil.

16) 1↔2, 2↔4, 3↔6, 4↔8, 5↔10, 6↔12, etc.

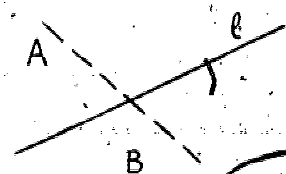
17) A↔BC
B↔AC
C↔AB

You could also have, A↔AB, B↔BC, C↔CA
A↔AC, B↔BA, C↔CB

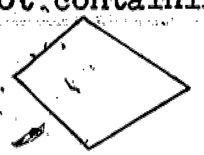
Exercises - pp. 15-17

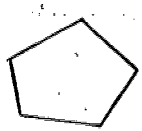
1) The set consisting of a point C on ℓ and between A and B.

2) The set consisting of a line ℓ intersecting line segment AB and not containing A or B.



3)


quadrilateral


pentagon


ellipse

4) The set consisting of points R, P, and U

5) The set consisting of points D and E

6) <u>The Point W</u> is the intersection set of the line segment MK and the line determined by points X and Y (or the ray through X with endpoint Y).

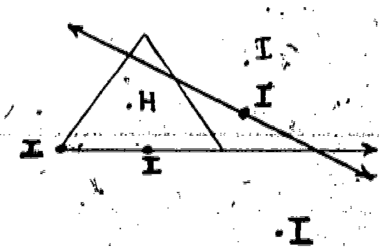
<u>The Point X</u> is the intersection set of the line segment KL and the <u>line segment</u> WY (or you could use ray through W with endpoint Y)
(or " " " " " Y " " W)
(or " " " line l determined by points W and Y)

<u>The Point Y</u> is intersection set of the ray through L with endpoint M and the ray through X with endpoint W.

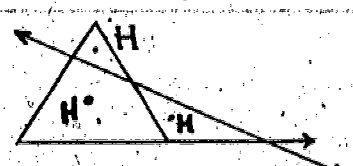
In all of above descriptions you could use half-lines in place of rays.

7) △ MKL, △ MWY, △ WKX, △ XLY

8)



9)



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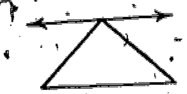
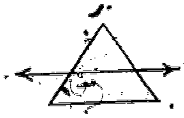
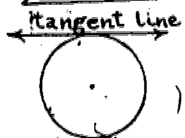
Exercises - pp. 15-17 cont.

10) impossible

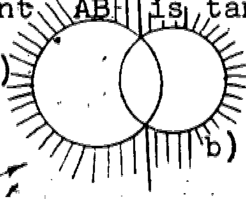
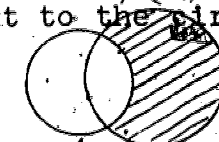
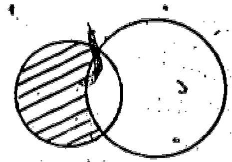
11) impossible

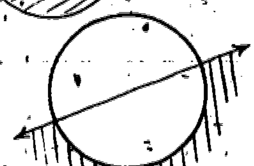
12) a)  b)  c)  d) impossible

13) a)  b) 

c)  d) impossible

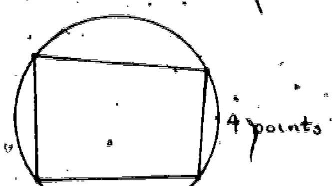
14) You must consider three situations similar to 13. a, b, and c, and the situation where one point lies inside and one outside or both inside the circle.
Yes, (if line segment AB is tangent to the circle)

15) a)  b)  c)  or 

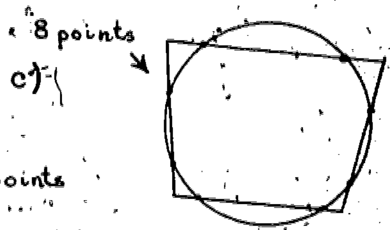
16) a)  b) 

16) c) 1.- the set of points outside the circle and in the half-plane containing A and 2.- the set of points inside the circle and in the half-plane not containing A.
We must also include the points of the circle and the line. These points were not shaded in a) or b) either.

17) a) one possibility is: b) one possibility is: c)

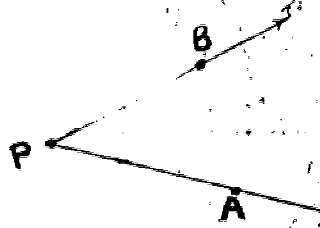


b) 

c) 

18) The set of points on the line - boundary line
The set of points in the half-plane on one side of boundary line
The set of points in the half-plane on the other side of boundary line.

19) The inside is the set of points in the intersection set of the two half planes determined by the lines on PA containing B and on PB containing A. The outside is the set of all other points except those on the angle.



Exercises - pp. 19-20

1) a) The intersection set would be straight line (horizontal) where the ceiling meets the front wall.

b) The intersection set would be the straight line (vertical) where the front wall meets the side wall.

c) Yes, the corner where the ceiling, front and side wall all intersect. This is the one common point.

2) a) The intersection set would be a point lying on both the line segment AB and the plane.

b) The null set or empty set

3) There are many pairs. One such pair would be the line which is the intersection set of the front wall and ceiling and the line which is the intersection set of a side and a back wall.

4) Revolving doors, pages in an open book

5) The intersection sets (lines) mentioned in 3. above furnish many examples. Buildings have numerous skew lines.

6) The empty or null set.

7) We might call the points lying outside the plane two half-spaces, and the points on the plane the points of the boundary plane.

9) The faces are sets of points which (if extended) determine planes. There are six of these planes.
The edges are sets of points which (if extended) determine lines There are 12 such lines.
These lines (or edges) are intersection sets of pairs of planes.
The corners (of which there are 8) are the intersection sets of three planes not all intersecting on one line. These intersection sets are points.

10) The faces are sets of points, each set (if extended) determining a plane. The planes determined by these sets of points intersect in pairs forming 6 intersection sets which are lines (if extended). The planes intersect in threes, determining 4 intersection sets which are points.

Exercises - pp. 22-23

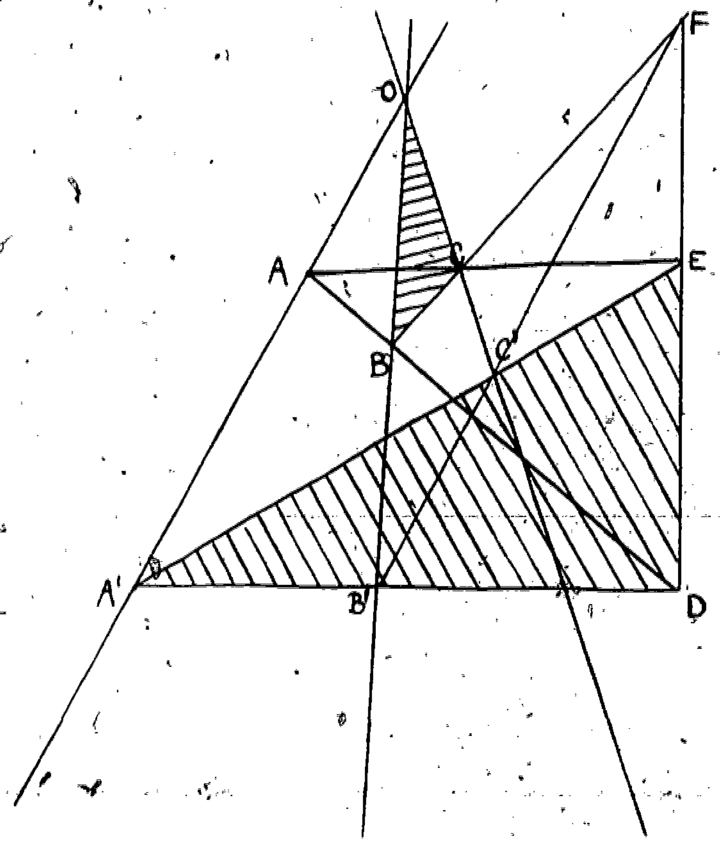
1) d) yes, D, E, and F are collinear

2) -

3) -

Exercises - pp. 22-23 cont.

4) See figure



5) From the figure above, we see that the vertices of $\triangle$ AA'E and $\triangle$ BB'F lie on the three lines passing through D. AA' meets BB' at O, AE intersects BF at C, and A'E intersects B'F at C'. As can be seen, O, C and C' lie on the same line (are collinear).

UNIT VI

Sample Test Questions

1. The set of points sketched below represents a (line).

2. The set of points sketched below represents a (ray).

3. The set of points RS represents a (line segment).

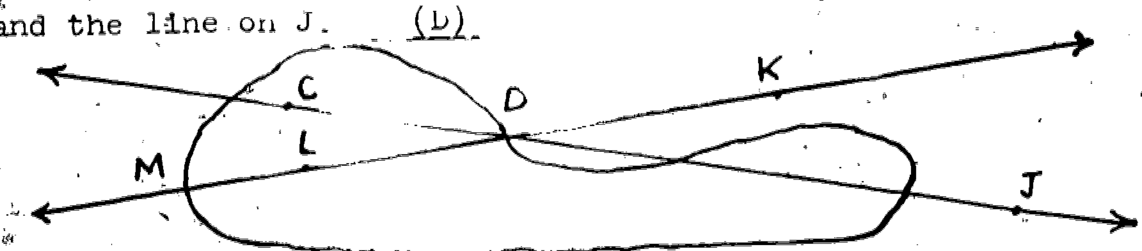
4. The set of points sketched below represents a (simple closed curve).

5. Set A consists of the elements 2, 4, 6, 8; set B consists of the elements 2, 6, 10, 14. What elements are in the intersection set of Set A and Set B? (2, 6)

6. A set must have at least two elements. True **False**

7. A line segment has two end points. **True** False

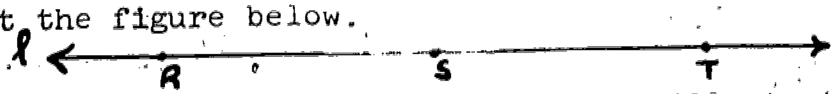
8. In the sketch below, name the intersection set of the line on L and the line on J. (D)



9. In the sketch above, name three line segments having L as end point. (LM) (LD) (LK)

10. The line on C separates the set of points on the paper into how many sets? (3)

11. Look at the figure below.



Find in column B the name of a set which illustrates each

item in Column A. Write its number in the space at the left.

		Column A	Column B
(6)	A.	line	1. R
(3)	B.	intersection of two rays with endpoint T.	2. RS
			3. T
(4)	C.	intersection of RS and RT.	4. ST
			5. R and T
(1)	D.	endpoint of a ray which contains T.	6. 1
			7. RT
(2 or 7 or 4)	E.	line segment.	8. The empty set.
(7 or 8)	F.	intersection of a ray with endpoint R and a ray with endpoint T.	
(5)	G.	endpoints of a line segment.	

12. The set of points on two rays with a common endpoint is called

 A. a triangle.

 *B. an angle.

 C. a vertex.

 D. a side of a triangle.

13. Which one of the following intersection sets is impossible for a line and a square?

 A. the empty set.

 B. a set of 1 element.

 C. a set of two elements.

 *D. a set of 3 elements.

14. The intersection set of two planes is

A. a point.

*B. a line.

C. a plane.

D. 2 points.

15. A line on a plane separates the plane into 3 sets which consist of

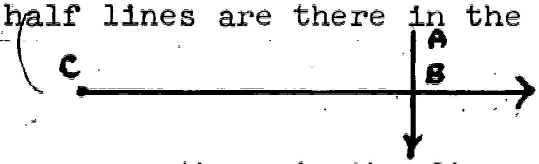
A. 3 planes.

B. 3 half planes.

*C. 2 half planes and a line.

D. 3 lines.

16. How many half lines are there in the figure below? (4)



17. How many rays are there in the figure above? (4)

18. How many line segments? (2)

19. It is <u>impossible</u> for the common intersection of three planes to have an intersection set that is

A. the empty set.

B. a set with one element.

*C. a set with two elements.

20. How many triangles does the figure contain?

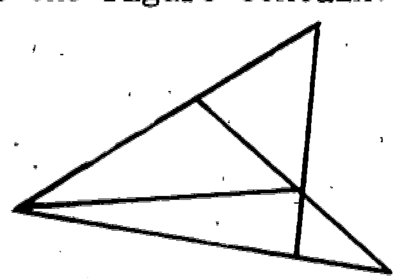
A. 4

B. 6

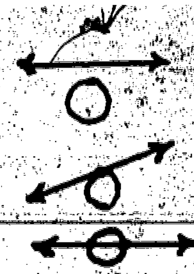
*C. 8

D. 10

E. 12



21. Make sketches in which the intersection set of a straight line and a circle is the set described. If it is impossible to draw such a sketch, write "impossible."



A. The empty set. (a)

B. A set of one element. (b)

C. A set of two elements. (c)

D. A set of 3 elements. (d) (Impossible)

Date ______________

UNIT: ______________

Name of Teacher: ______________

Name of School: ______________

City: ______________ State: ______________

Number of days given to the teaching (including testing) of this unit: ______________

Approximate dates: ______________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?

   Did you omit any part? ________

4. Did you use any supplementary developmental materials? ______

   If so, what were they, and at what points were they used?

5. Did you find it necessary to provide the pupils with additional material? ________

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades? ________

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

Summary of Teachers' Comments

The number of days spent on this unit varied from 10 to 22. Most of the pupils seemed to be in the more able group, although some classes were reported as medium or low in ability. Five teachers omitted no part of the unit. A majority felt that the unit included sufficient developmental and practice material. The teachers were almost in complete agreement that the unit should be included in the seventh grade curriculum.

Opinions concerning the individual sections follow:

Section	Easiest to Teach	Most Difficult to Teach
Sets and points on a line	10	1
Intersection	5	3
Line segments, half-lines, and rays	11	3
Lines on a point	0	2
One to one correspondence	0	6
Closed curves	4	1
Quadrilateral	0	2
Planes and skew	0	14
Special figures	0	18
Whole unit	11	0
No parts of unit	0	6
Angle	0	3

Comments of 18 teachers (23 classes) on Unit VI (Non-Metric Geometry) fall into two patterns as indicated by the following abbreviations: "Understandable and clear"; "a good unit"; "refreshingly different"; "interesting to teach"; "students enjoyed the unit"; "both exciting and frustrating"; "best adapted to better students"; "More developmental material needed"; "some parts too difficult"; "lacked motivation"; "students had considerable difficulty".

Other comments in the nature of advice to other teachers include:

Students and teacher learn together.

Students with I.Q. of 75 learned some of the definitions and participated in the class exercises.

An additional helpful reference is: Hardy and Wright. Theory of Numbers.

Students who are unfamiliar with the idea of set need a little more work with "sets" prior to the introduction of "intersection sets."

Pupils may show a preference for traditional material unless the teacher is conscious of his responsibility for maintaining interest initially obtained by the "differentness" of the material.

Do not hurry the class, particularly in the first parts of the unit. Pupils need time to develop concepts.

Interest in the Desargues Configuration may be difficult to obtain.

UNIT VIII

INFORMAL GEOMETRY I

By "informal geometry" we mean the study of the properties of geometric figures through experimenting, measuring, and formulating conclusions by inductive reasoning. This is in contrast to the development of a logical organization of theorems about the properties, by deductive reasoning from undefined terms and stated assumptions. In this unit, in addition to experimenting, measuring, and inductive reasoning, the pupils are introduced to the use of deductive thinking as a method of reaching conclusions, arguing from previously stated principles and definitions. We reserve for later the systematic organization of geometry as a deductive science, starting with postulates and undefined terms, and developing theorems and definitions on this basis.

The purposes for which the unit was planned were these:

To develop awareness of the occurrence in the environment of illustrations of points, lines, and planes, and also of three-dimensional regions bounded by points, lines, and planes.

To introduce certain geometric concepts and relations.

To give the pupils experience in discovering empirical laws about spatial relations on the basis of systematic observations of geometric figures.

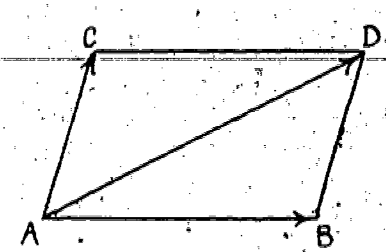
To give the pupils experience in verification of experimental results and in informal deductive argument on the basis of previously stated principles.

The major topics are (1) the angle relationships in the figure formed by parallel lines and a transversal, and (2) angle relationships in triangles. It is assumed that the pupils are already familiar with the concept of angle, conventional angle notation, angle measure-

ment, and the use of a protractor. In addition, they should have studied the unit on non-metric geometry in this series. The concepts developed in that unit used most frequently in this one are those of point, line, ray, line segment, plane, and half-plane. Some notation introduced in that unit is also used in this one.

The introductory section serves as a point of departure for the discussion of physical representations of geometric concepts. Discussions of this nature should take place frequently. Whenever a new arrangement of points and lines is studied it is helpful to ask the pupils to find illustrations in the environment or in pictures. In the first discussion, a chalk box and other solids may be used to clarify notions of plane surfaces which intersect or do not intersect, lines which do or do not intersect, and the like.

We also give some hints about applications for the purpose of motivation. You may supplement these remarks by pointing out the geometrical knowledge involved in designing an airplane. The main problem is to find out how air will flow about an airplane of a given shape moving in a given direction at a given speed. From this we can calculate the lifting force and the air resistance. You may also use the parallelogram of forces as an illustration. In order to find the single force equivalent to two given forces acting simultaneously at a point we can draw a diagram like this,



in which the given forces are represented in magnitude and direction by the segments AB and AC. We complete the parallelogram, and the diagonal

AD gives the magnitude and direction of the resultant force. Geometry is also used to figure out the forces in an electromagnetic field, and why rubber is elastic, and how an oil company should schedule its production. In the theory of relativity and in the design of agricultural experiments completely different concepts of space are used. Nowadays the physicist, the chemist, the biologist, the engineer, the economist, the psychologist, and the military strategist use geometry in ways far removed from surveying, some of which were not even dreamed of 15 years ago.

The historical remarks serve both as motivation and also to give the children a feeling for mathematics as a living, growing science, and as an integral part of our culture. The pupils should look up the names in books and encyclopedias and their reports should be posted where all can refer to them. The annual supplements to some of the encyclopedias often list, under the heading of "Mathematics", important recent developments. These are usually too technical for the children, but can be used to show who is doing outstanding work today.

The Scientific American often publishes excellent expository articles, and recently several popular magazines such as Fortune, Reader's Digest, and Esquire have carried articles on mathematics and mathematicians.

Part A provides background for the study of a transversal intersecting parallel lines, and of triangles. It deals with three lines which intersect in one, two, three, or no points, and the associated angle relationships. The concepts of vertical angles and of adjacent angles are introduced, and are defined by using the concepts of ray and half-plane which were developed in the unit on non-metric geometry.

Observe our careful use of language. We adhere to the uniform usage of "a = b" to mean that a and b are the same thing. Different angles or segments may have the same measure, and they will then be said to be equal in measure. Later we shall call such figures congruent. At this point it is more important to instill in the pupils good language habits than to spend much time on a formal discussion of linguistic matters.

The equality of measures of vertical angles is developed in two ways. First the pupils are given exercises in drawing two intersecting lines and in measuring the angles formed. By examining the measures obtained and then comparing their results with those obtained by other pupils they should arrive at a tentative conclusion. The figure is then re-examined, certain properties of adjacent angles are noted, and the same conclusion is reached by deductive reasoning. The use of data from measurements as a basis for a tentative conclusion, followed by informal deductive argument, occurs at several other points in the unit.

The emphasis should be on the children's discovering the relationships by their own observations. Be careful not to kill their interest by telling them the answers. Be patient and, if necessary, ask leading questions, but try to draw the answers out of the students. It will be more stimulating to the whole class if one of them makes the discovery instead of your telling them. You should guide the class discussion so as to make the children realize the necessity of saying exactly what they mean. Have the youngsters criticize each other's formulations of the generalizations until they arrive at a statement of each principle which satisfies all. Then you may show them our formulation for

comparison. They may then use these statements as models in their further work on this unit. When they have finished the unit, you should distribute our formulation of the principles so that the students may insert them in their notebooks for future reference. The practice in stating a proposition clearly and precisely is important for all future work.

A procedure which is helpful in guiding pupils in formulating tentative conclusions on the basis of experimentation and measurement may be illustrated in connection with Exercise 5 in Part A. As the pupils are measuring their angles, a table may be laid out on the blackboard, with the names of the pupils in the first column, and the names of the angles measured at the top.

pupil	Angle BAD	Angle CAE	Angle DAC	Angle BAE

As each pupil completes his measuring, he can report his results, or record them himself in the table. After a few pupils have reported, the class can observe the relationship between the measurements recorded in the table, and note that while the figures reported by different pupils are not the same, the measurements obtained by each pupil for each pair of vertical angles are the same, or nearly so. The observation of the measurements obtained by several pupils supports the conclusion about the equality of the measures of vertical angles, and the procedure suggests to the pupils a method of organizing data of this kind.

In Part B, the set of three lines in which two lines (not necessarily parallel) intersect a transversal is studied. The angle pairs identified earlier (vertical, adjacent) are noted, and the new angle pairs (corresponding, alternate interior) are defined. While studying

Part B the pupils should become well acquainted with this figure, so that they can readily recognize each kind of angle pair.

In Exercise 3, the symbol used to denote equality of measures is explained. We have used the symbol " $\stackrel{m}{=}$ " when the measures of two angles are equal. "$\angle a \stackrel{m}{=} \angle d$" can be read, "The measure of angle a equals the measure of angle d."

The discussion in the paragraph between exercise 5 and 6 may require review of the concept of half-plane. Recall that a line (unlimited in length) is said to divide the plane (also unlimited in extent) into two half-planes, the line being the boundary between the two half-planes.

In Part C, the relation between the measures of the angles which two lines form with a transversal and the intersection or non-intersection of two lines is explored through measurement of angles. The tentative conclusion is formulated, and accepted, that when a pair of corresponding angles have the same measure, the lines do not intersect and are parallel. This yields the construction for parallel lines which is used in the unit, with a second proposed in exercise 8. In Part D, informal deduction is used to establish the principle that when a pair of alternate interior angles have the same measure, the lines are parallel.

In the Exercises following Part D, pupils may have difficulty with exercises 4-5. One way to help them to recognize a familiar figure within a more complex one is to cover up one of the transversals and extend the remaining transversal and segment RS.

In exercise 6, if a T-square or plastic triangle is available, either may be used to demonstrate the application of Principle 2.

Part E calls attention to a common fallacy, which is to assume that if a statement is true, its converse is true also. It will be profitable to discuss many illustrations to show that the converse of a true statement may be true or may be false.

In Part F, the converses of Principle 2 and Principle 4 are investigated. The converse of Principle 2 is stated as a principle, on the basis of measurement, and the converse of Principle 4 is then argued deductively. Some of the exercises give experience in identifying angle pairs in more complex figures.

In Part G, the angle relationships in a triangle are studied. Exercises are designed to lead to the discovery of the angle-side relationships in an isosceles triangle, from which the corresponding relationships in an equilateral triangle may be deduced. In exercise 3 it would be well to have the pupils use compasses to draw equilateral triangles, if the compasses are available. Exercises 10-11, and also exercises 12-13, provide an opportunity to note that one conclusion is the converse of the other.

In Part H, the angle-sum for a triangle is developed, first experimentally and then deductively by the use of parallel lines. A few exercises are provided to show the significance of this principle, but additional ones may well be used.

Throughout the study of this unit effort should be made to encourage the pupils to discover relationships for themselves, both by measurement and observation of results, and by informal deduction, and to state clearly, in their own words, what their conclusions are and their reasons. Some pupils will find the exercises much easier than others find them. Some will need help in reading and following directions, and some will have keener space perception than others. It is probable

that a few pupils could take the problem of discovering angle relationships among three lines, and, with relatively little guidance, discover most of the relationships included in the unit.

ANSWERS

Introductory section

1. Illustrations of planes -- desk tops, floors, windows, book covers, wall picture.
2. Illustrations of lines which are intersections of planes -- walls, wall and ceiling, walls and floor, book pages, hard cover notebook standing on the desk, surfaces of filing cabinet, desk surfaces.

   Illustrations at points which are intersections of lines -- corners of room, edges at windows, lines on note paper, edges of books.
3. Illustrations of three planes intersecting in a line -- 3 adjacent pages of a book, three cards attached with adhesive tape.
4. Illustrations of parallel planes -- opposite walls of room, opposite sides of desk top, covers of book, two surfaces of window pane.

   Illustrations of parallel lines and skew lines -- opposite edges of room, opposite sides of sheet of paper, edges of papers in a haphazard pile, intersection of two walls and that of a third wall with floor.
5. Illustrations of curved surfaces -- pipes, light reflectors, chair surfaces, curled paper, clock face glass, globe, lenses of glasses.
6. Illustrations of planes on the street -- sides and roofs of buildings, sidewalks windows, billboards. At least six planes are needed to enclose a house. Example of curved surfaces -- car bodies, curved windows, light poles.
7. From the Greek words meaning "earth" and measurement".

Part A. Sets of Three Lines

1. Two straight lines intersect in at most one point.

2. Four angles are formed by two intersecting straight lines.

3. Angle BAE and angle DAC are also vertical angles.

   Angles BAD and DAC are adjacent angles.

4. Vertical angles have the same measure.

5. Each pair of vertical angles have the same measurement even though the size of the angles in the figures vary.

6. (a) 180° (b) 180° (c) 50°, 50°

   (d) $x + y = 180$, $z + y = 180$

   $x = 180 - y$, $z = 180 - Y$.

   (e) $x = z$. Yes.

8. Six angles in all -- angle BAD, DAF, FAC, CAE, EAG, GAB. All have a common vertex A.

9. Three pairs of vertical angles -- BAD and CAE, DAF and GAE, FAC and BAG.

10. 360°

11. Three angles lie on one side at ℓ_1. The sum of their measures is 180°.

12. Yes.

15. Triangle.

16. Each vertex has four angles. Angle pairs with a common vertex are vertical or adjacent.

18. 4 pairs of vertical angles.

    8 pairs of adjacent angles.

19. The figures are alike in that they are all formed by 3 lines in one plane.

The figures are different in the relationships between pairs of lines.

The three lines may be parallel.

20. One point, two points, three points or no points.

22. The lines may be skew, that is they may not lie in one plane.

23. The lines may be skew to each other and may have no intersections. Two may intersect and both be skew to the third, yielding only one intersection. Two may be skew and the third may intersect both forming two intersections. These may be illustrated with sticks or curtain rods.

Part B. Two Lines and a Transversal

1. Figures for problems 14 and 17.

2. Problem 14 --

m_1 intersects m_2 and m_3

m_2 intersects m_1 and m_3

m_3 intersects m_1 and m_2

Problem 17 --

t_3 intersects t_1 and t_2

3. 8 angles, 4 pairs of vertical angles, angle a $\stackrel{m}{=}$ angle c, angle d $\stackrel{m}{=}$ angle b, angle e $\stackrel{m}{=}$ angle g, angle f $\stackrel{m}{=}$ angle h.

4. Angle a and angle b, angle b and angle c, angle c and angle d, angle d and angle a, angle e and angle f, angle f and angle g, angle g and angle h, angle h and angle e.

5. All the adjacent angle pairs of number 4.

6. Angle a and angle e, angle c and angle g, angles b and f, angles d and h.

7. Angle d and angle f, angle c and angle e.

8. The four pairs of vertical angles in Example e, by Principle 1.

9. a. Angle v

   b. Angle s

   c. Angle w

   d. Angles r and p

10. a. Angles r and p, angles x and v, angles s and w, and angles z and y.

    b. Angle r and angle z, angle x and angle s, angle v and angle w, angle p and angle y.

    c. Angle x and angle w, angle p and angle z.

    d. Angle r and angle x, angle x and angle p, angle v and angle p, angle r and angle v, angle z and angle s, angle s and angle y, angle y and angle w, angle w and angle z.

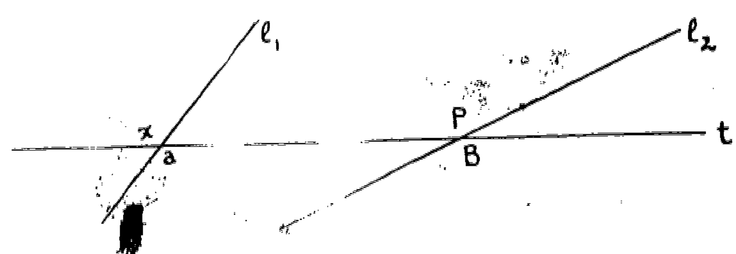
Part C. Parallel Lines (Corresponding angles)

1. Yes. On the same side of t as the two angles.

2. The roads will intersect on the left side of t.

4. Corresponding angle.

7.



a. Below t b. Parallel c. Above t d. Below

e. Parallel f. Above g. Below h. Parallel

i. Above j. Parallel k. Below l. Above

8. The distances between the intersections are equal. a. If ℓ (CD) is the length of C D, then $\ell(CD) = (\frac{\sin 40^\circ}{\sin y})\,\ell(AB)$.

This formula is for the information of the teacher.

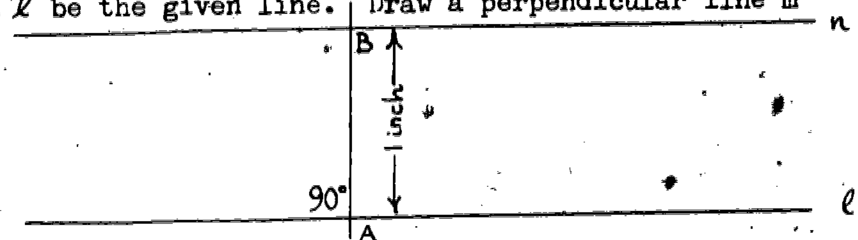
Measure of Angle y in degrees	$\frac{l(CD)}{l(AB)}$
10	3.7
20	1.9
30	1.3
40	1.0
50	.84
60	.74
70	.68
80	.65
90	.64
100	.65
110	.68
120	.74
130	.84
140	1.0
150	1.3
160	1.9
170	3.7

The distance from C to D depends only on the angle y.

The minimum distance occurs when angle y $\stackrel{m}{=}$ 90°.

10. If ℓ(AB) is doubled, then ℓ(CD) is also doubled.

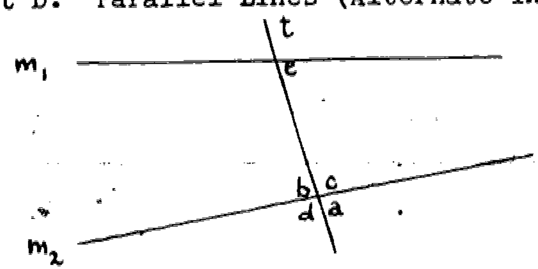
11. Let ℓ be the given line. Draw a perpendicular line m



Mark off a point B on m whose distance from the intersection A of ℓ with m is one inch. Draw the line n through B perpendicular to m.

Part D. Parallel Lines (Alternate-Interior Angles)

1.



2. Angle c $\stackrel{m}{=}$ 150°, angle d $\stackrel{m}{=}$ 150°, angle a $\stackrel{m}{=}$ 30°, Principle 1.

3. Angle a.

4. Yes, to the right of t.

5. If angle b is larger than angle c, then m_1 and m_2 intersect on the same side as angle e, but if angle e is larger, then they intersect on the same side as angle b, by principles 1 and 3. If angles b and e have the same measure, then m_1 and m_2 are parallel, by principles 1 and 2.

8. a. Parallel, 4 b. Parallel, 2 c. Above t, 3 d. Below t, 2* or 4*
   e. Parallel, 2* or 4* f. Above t, 2* or 4* g. Above t, 2*
   h. Above t, 1 and 2 i. Above t, 4 j. Parallel, 2

Exercises

1. Vertical, adjacent, corresponding, alternate interior

2. a. a and d or b and e or c and f

   b. a and b, b and c, c and d, d and e, e and f, f and a

   c. a, b, and c, or b, c, and d, or c, d, and e, etc.

3. a. a and g, b and h, c and e, f and d.

   b. a and h, h and g, g and b, b and a, c and f, f and e, e and d
      d and c

   c. same as b.

   d. a and c or b and d or h and f or g and c

   e. b and f or c and g

4. a. Angle $x \stackrel{m}{=} 75^\circ$, by principles 4 and 5

   b. Angle $z \stackrel{m}{=} 45^\circ$, by principle 4. (If angle z has some other measure, then ℓ must intersect R S.)

   c. 180°

   d. 60°

5. a. 180°

   b. 65°

   c. BC is parallel to DE by principle 2.

*In these cases, use is made of the principle that adjacent angles are supplementary.

6. b. Principle 2

Part E. Turning a Statement Around (Converse)

3. a. True; converse may be false.

   b. May be false; converse true.

   c. True; converse may be false.

   d. May be true or false (Mary may not go to school); converse also undetermined.

   e. True; converse also true.

4.

	Statement	Converse
a.	T	?
b.	?	T
c.	T	?
d.	?	?
e.	T	T

Part F. Converses of Principle 2 and Principle 4.

2. Corresponding angles or alternate interior angles.

3. Corresponding angles should have the same measures.

4. Corresponding angles should have the same measures.

5. Angle pairs y and u, x and s, z and w, w and r.

6. Angle pairs z and s, n and u.

7. Angle z $\stackrel{m}{=}$ angle w - corresponding angles

   Angle w $\stackrel{m}{=}$ angle s - vertical angles

   Angle z $\stackrel{m}{=}$ angle s - both have the same measure as angle w.

9. Angle pairs b and c, g and h.

10. Angle pairs a and h, b and e, f and c, g and d -- corresponding angles.

11. Angles a, c, d and f $\stackrel{m}{=}$ 68°

Angles b, e, and g $\stackrel{m}{=}$ 112°

12. 2

13. Alternate interior.

14. Corresponding angles, alternate interior angles, and vertical angles.

15. 2 Corresponding angles at x and s, R and W.

16. Angle a + angle b $\stackrel{m}{=}$ 180° since ℓ_1 is a straight line

angle b $\stackrel{m}{=}$ angle c - alternate interior angle

angle a + angle c $\stackrel{m}{=}$ 180° = angle b + angle d.

17. Same as number 16.

18. ℓ_2 is perpendicular to t

19. a. By the definition of perpendicular line

b. Corresponding angles have the same measure.

c. Angle b has the same measure as angle a.

d. By the definition of perpendicular lines.

20. a. 4 transversals. c (and d) intersects a and b and a (and b) intersects c and d. a and b are parallel lines and c and d are parallel lines.

b. Corresponding angle pairs are: (e,g), (f,h), (l,j), (k,i), (m,o), (n,p), (t,r), (s,q), (e,m), (f,n), (g,o), (h,p), (l,t), (k,s), (j,r), (i,q).

c. Alternate interior angle pairs are: (f,j), (k,g), (n,r), (s,o), (l,n), (k,m), (j,p), (i,o).

d. Vertical angle pairs are: (e,k), (f,e), (g,i), (j,h), (m,s), (n,t), (o,q), (p,r).

e. Angle pairs whose measures have a sum of 180° are: (e,h), (f,g), (l,i), (k,j), (m,p), (n,o), (t,q), (s,r), (e,t), (l,m), (f,s), (k,n), (g,r), (j,o), (h,q), (i,p).

f. Angles f, l, h, j, n, t, p, r each $\stackrel{m}{=}$ 28°.

Angles k, g, i, m, s, o, q each $\stackrel{m}{=}$ 152°.

Part G. Triangles

1. b and e are triangles. f contains a triangle.

   a has only 2 line segments.

   c and d are enclosed by 4 line segments.

5. a appears equilateral

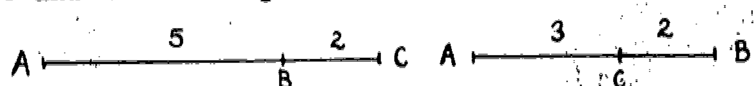
   d appears isosceles

   b, c, and e appear scalene

7. Yes

8. 1, 6, 7, and 1, 6, 9. The sum of the length of two sides of a triangle is greater than the length of the third side.

9. The sum of the measures of any two sides must always be greater than the measure of the third side. The length of AC must be less than 7 inches and more than 3 inches.



12. Angle a $\overset{m}{=}$ angle b, angle b $\overset{m}{=}$ c, thus all three angles have the same measure.

13. If angle a $\overset{m}{=}$ angle b, then sides opposite these angles have the same measure.

    If angle b $\overset{m}{=}$ angle c, then sides opposite these angles have the same measure.

    Thus all three sides have the same measure.

14. No.

15. The angle opposite the larger side has the larger measure. If the length of AC alone is doubled, then the angle ABC does not change in a simple way. If all three sides are doubled, the angles of the triangle do not change in measure.

Part H. Angles of a Triangle

1. The sum of the measures of the angles of a triangle is 180°.

3. Angles EAB and ABC are alternate interior angles, and are therefore equal in measure. Angles DAE and ACB are corresponding angles and are also equal in measure. The sum of the measures of angles DAE, EAB, and BAC is 180° since DC is a straight line. Therefore the sum of the measures of the angles of the triangle is 180°.

4. Angles FAC and ACB have equal measures by principle 6.

5.

	Desired part	Principle
a.	80°	10
b.	70°	10
c.	130°	10
d.	30°	10
e.	91°	10
f.	40°	8
g.	60°	8, 10
h.	4 in.	9
i.	4 in.	10, 9
j.	3 in.	10, 9

6. Either 50° and 80° or 65° and 65°.

7. $\angle ABC \stackrel{m}{=} \angle DEF$, by principle 10.

UNIT VIII

Sample Test Questions

PART I. TRUE - FALSE

F 1. Little has been added to geometry since it was invented.

F 2. If one angle of an isosceles triangle is equal to the measure of 66°, one of the other two angles must be equal to the measure of 66°.

T 3. A statement may be true while its converse is false.

T 4. Equal pairs of corresponding angles are formed when a transversal intersects two parallel lines in the same plane.

T 5. A statement and its converse may both be true.

F 6. The intersection set of three lines in a plane must be three points.

F 7. Many modern geometry textbooks are based on one written by Euclid.

T 8. If a triangle has two equal sides, it has two equal angles.

F 9. All isosceles triangles have the same shape regardless of size.

T 10. The sum of the measures of the three interior angles of a triangle is equal to the measure of 180°.

F 11. An equilateral triangle is also a scalene triangle.

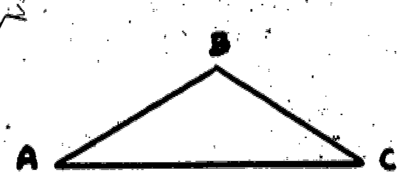
F 12. The converse of a false statement is always false.

F 13. If a triangle has only two equal sides, it can have three three equal angles.

T 14. An equilateral triangle is also an isosceles triangle.

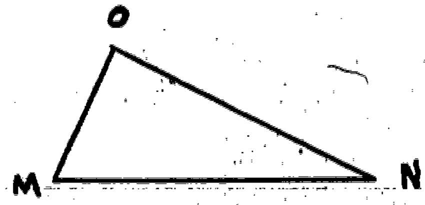
F 15. If a figure is a closed curve, it is a circle.

T 16. In the figure at the right, A, B, and C are symbols for the verticies of the triangle.



F 17. If a transversal intersects any pair of lines in the same plane, the alternate interior angles are equal.

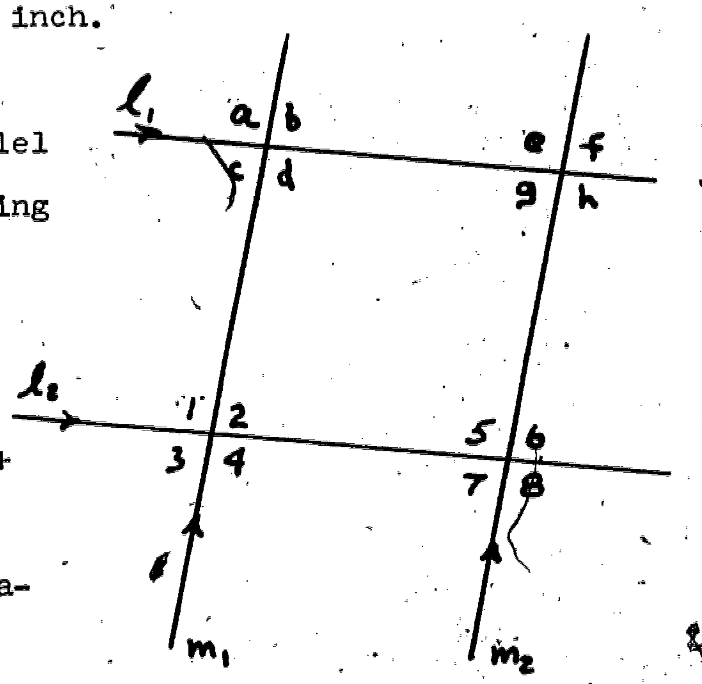
F 18. In the figure shown at the right, if all measures are held equal except angle MON and side MN, as angle MON increases in size the length of MN will decrease.



T 19. One method of proof is to reduce a statement to a previously proved statement.

F 20. It is possible to draw a triangle whose sides measure 4 inches, 2 inches, and 1 inch.

The figure shown at the right consists of two pairs of parallel lines. Use the figure in marking 21 - 26 true or false.



T 21. Angle a $\stackrel{m}{=}$ angle 8.

F 22. Angle 3 $\stackrel{m}{=}$ angle h.

F 23. The measure of angle a + measure of angle d = measure of angle 3 + measure of angle 2.

T 24. The measure of angle c + measure angle h = measure of angle 2 + measure of angle 5.

T 25. The figure contains more than 16 pairs of equal angles.

T 26. If angle 7 $\stackrel{m}{=}$ angle 8 then all the measures of the angles

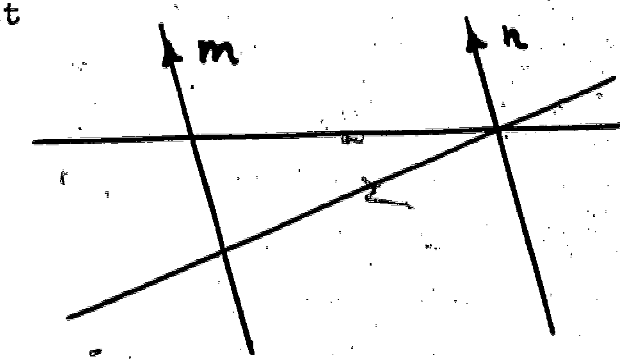
shown in the figure are equal.

PART II. MULTIPLE CHOICE.

1. If a transversal intersects two lines in the same plane and the measures of the corresponding angles are equal, then the two lines are...
   *A. parallel lines.
   B. skew lines.
   C. perpendicular lines.
   D. intersecting lines.
   E. none of the above answers is correct.

2. If one angle of a scalene triangle $\overset{m}{=}$ 50°, which of the following statements is always true?
   A. One of the other angles $\overset{m}{=}$ 90°.
   B. One of the other angles $\overset{m}{=}$ 50°.
   *C. The sum of the measures of the other two angles is 130°.
   D. Two of the sides are equal.
   E. One of the other angles $\overset{m}{=}$ 130°.

3. In the figure shown at the right, how many transversals intersect lines m and n?



   A. 1
   *B. 2
   C. 3
   D. 4
   E. 5

4. If the measure of one angle of a triangle is equal to the measure of another angle in the triangle...

A. the measures of the sides of the triangle are equal.

B. none of the measures of the sides are equal.

C. the measures of the sides opposite the angles whose measures are equal are not equal.

*D. the measures of two sides are equal.

E. none of the above statements is correct.

5. If two sides of a triangle measure three inches and four inches, the third side could measure...

A. one inch.

B. seven inches.

C. less than one inch.

*D. more than seven inches.

E. none of the above answers is correct.

6. If three lines are drawn on the same plane and none of these lines are parallel, the figure formed could include...

A. exactly three angles.

B. exactly two points of intersection.

*C. a triangle.

D. two closed curves.

E. a rectangle.

7. A distinct point is...

A. a point you can see.

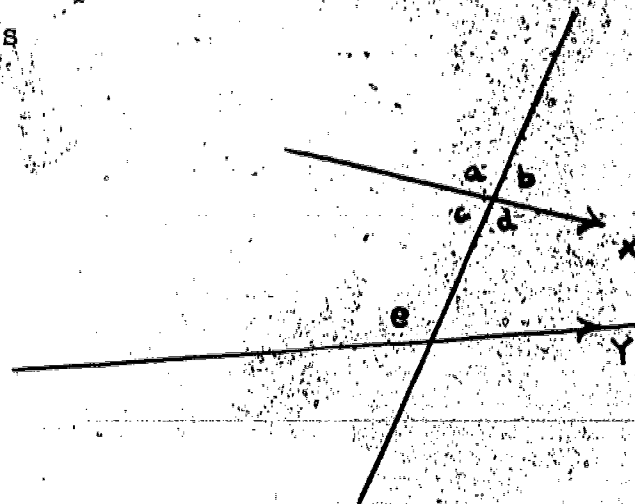
B. a sharp object.

C. an important statement.

D. a circular mark made with a pencil.

*E. the intersection of two lines.

8. In the figure on the right, the transversal t intersects x and y. Which angle forms with e a pair of alternate interior angles?

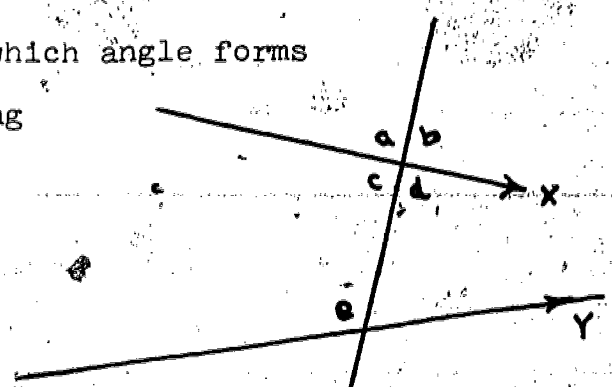


A. a
B. b
C. c
*D. d
E. e

9. A geometric plane is...
A. an airplane.
B. a carpenter's tool.
*C. a flat surface like that of a window pane.
D. a curved surface like that of a round lamp shade.
E. an object like a sheet of plywood.

10. A surface is...
A. a geometric figure.
*B. a set of points.
C. a set of lines.
D. a set of lines and points.
E. a measure of area.

11. In the figure on the right, which angle forms with e a pair of corresponding angles?



*A. a
B. b
C. c

D. d

E. e

12. In the figure above, lines x and y are parallel if...

A. angle a has the same measure as angle d.

B. angle c has the same measure as angle e.

C. angle b has the same measure as angle d.

*D. angle e has the same measure as angle d.

E. angle a has the same measure as angle b.

13. In the figure above, if the measure of angle e is 100°, the measure of angle c is...

A. 100°

B. 80°

C. 20°

D. 10°

*E. unknown.

14. In the figure above, an angle adjacent to angle a is...

*A. b

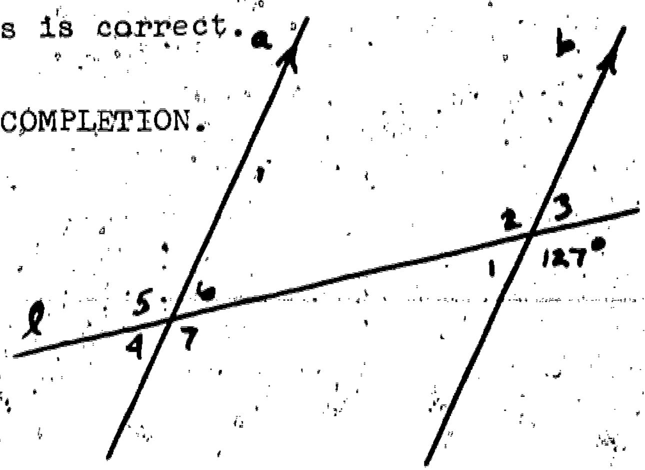
B. d

C. e

D. angles b, c, and d are all angles adjacent to angle a.

E. none of the above answers is correct.

PART III. COMPLETION.

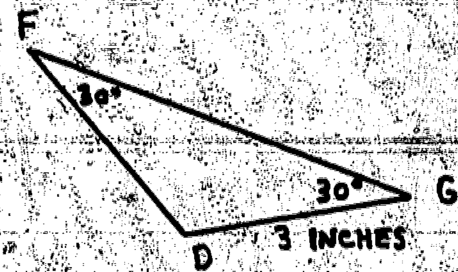


1-4 In the figure at the right lines a and b are parallel and intersect the transversal l. What are the measures of each of the follow-

ing angles? Note: the measures of one of the angles is given in the figure.

1. angle 6 (53°)
2. angle 2 (127°)
3. angle 1 (53°)
4. angle 5 (127°)

5. In the figure at the right line DF $\overset{m}{=}$ (3").

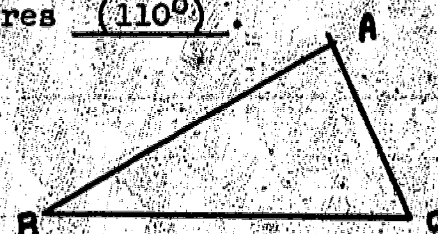


6. One of a pair of vertical angles measures 40°; the other one measures 40°.

7. If 3 lines intersect on a point and all measures of angles formed are equal, what is the measure of each angle? (60°)

8. Two lines intersect on point A. If the measure of one angle formed is 70°, an adjacent angle measures (110°).

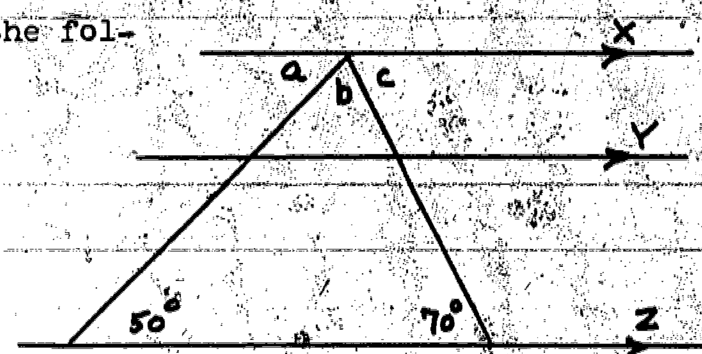
9. In the triangle at the right, angle ABC $\overset{m}{=}$ 30°. Angle BCA $\overset{m}{=}$ 70°. What is the measure of angle CAB? (80°)



10. Corresponding angles are on (same) side of the transversal.

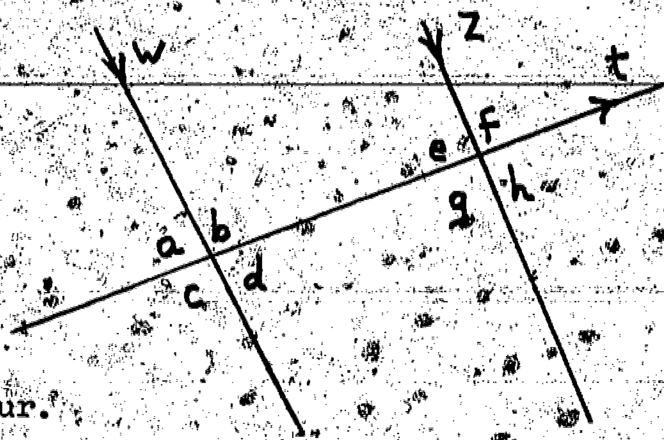
11 - 15 In the figure at the right, lines x, y, and z are parallel. What is the measure of the following angles?

11. angle a (50°)
12. angle b (60°)
13. angle c (70°)
14. angle d (130°)
15. angle e (50°)



16) When a line intersects two other lines in distinct points, it is called a (an) (transversal) of those lines.

17 - 20 Using the figure at the right, predict whether lines w and z will be parallel or will intersect. If they intersect, indicate which side of t (above t or below t) the intersection will occur. Fill in the space which correctly completes the following statements.



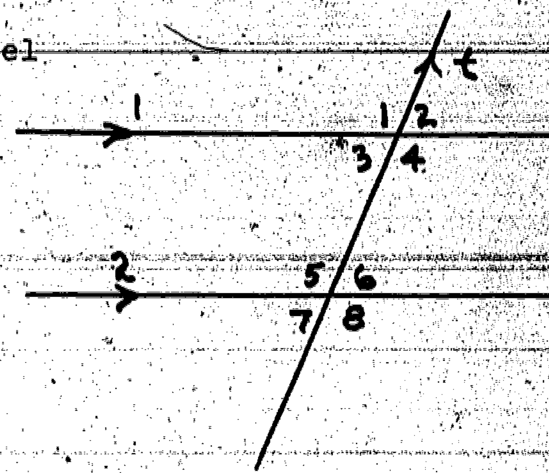
	intersect above t	intersect below t	be parallel
17. If angle a $\stackrel{m}{=}$ 75°, and angle e $\stackrel{m}{=}$ 75°, the lines will...	( )	( )	*( )
18. If angle b $\stackrel{m}{=}$ 100°, and angle e $\stackrel{m}{=}$ 80°, the lines will...	( )	( )	*( )
19. If angle c $\stackrel{m}{=}$ 120°, and angle g $\stackrel{m}{=}$ 100°, the lines will...	( )	*( )	( )
20. If angle d $\stackrel{m}{=}$ 60°, and angle e $\stackrel{m}{=}$ 80°, the lines will...	( )	*( )	( )

PART IV.

For each of the following select the statement which contains the minimum or least amount of essential (necessary) information

to prove the conclusions listed or to complete a true statement.

1. In the figure at the right, lines 1 and 2 intersected by the transversal t are parallel lines if...



*A. angle 5 $\overset{m}{=}$ angle 4.

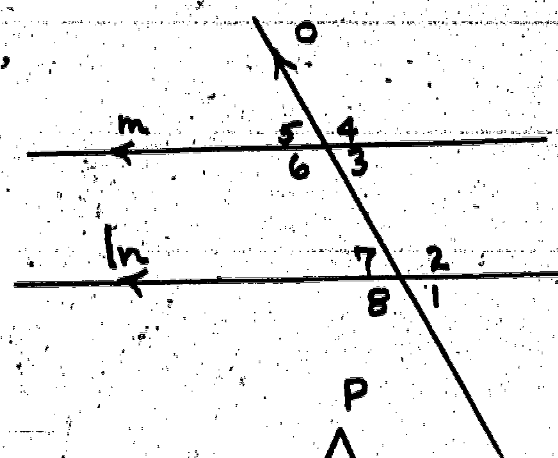
B. angle 3 $\overset{m}{=}$ angle 4.

C. angle 7 $\overset{m}{=}$ angle 6 $\overset{m}{=}$ angle 3 $\overset{m}{=}$ angle 2.

D. angle 4 $\overset{m}{=}$ angle 3 $\overset{m}{=}$ angle 6.

E. the sum of the measures of angles 2 and 4 equals the sum of the measures of angles 7 and 8.

2. In the figure at the right, line o intersects lines m and n. Angle 5 $\overset{m}{=}$ angle 7 if...



A. angle 5 $\overset{m}{=}$ angle 3.

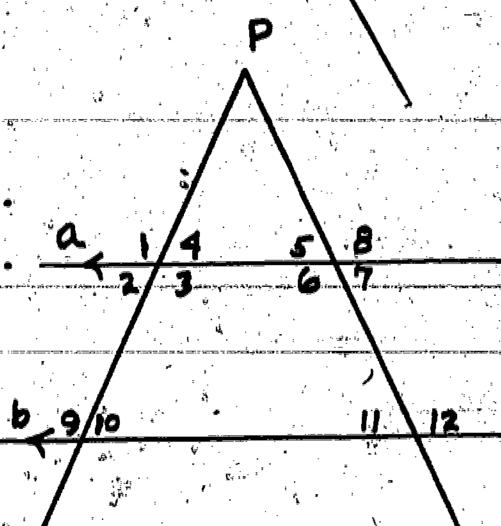
B. angle 2 $\overset{m}{=}$ angle 8 $\overset{m}{=}$ angle 7 $\overset{m}{=}$ angle 4.

C. angle 7 $\overset{m}{=}$ angle 2.

D. angle 3 $\overset{m}{=}$ angle 4 $\overset{m}{=}$ angle 5.

*E. angle 4 $\overset{m}{=}$ angle 2 $\overset{m}{=}$ angle 8.

3. In the figure at the right, angle 10 $\overset{m}{=}$ angle 11, and lines a and b are parallel. Angle 7 $\overset{m}{=}$ angle 10 because...



A. angle 4 $\overset{m}{=}$ angle 5.

B. angle 5 $\overset{m}{=}$ angle 7 $\overset{m}{=}$ angle 11.

C. angle 6 $\overset{m}{=}$ angle 12.

*D. angle 10 $\overset{m}{=}$ angle 11 $\overset{m}{=}$ angle 7.

E. angle 10 $\overset{m}{=}$ angle 2 $\overset{m}{=}$ angle 4.

Date ____________________

UNIT: ____________________

Name of Teacher: ____________________

Name of School: ____________________

City: ____________ State: ____________

Number of days given to the teaching (including testing) of this unit: ____________

Approximate dates: ____________________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?

   Did you omit any part? ________

4. Did you use any supplementary developmental materials? ______

   If so, what were they, and at what points were they used?

5. Did you find it necessary to provide the pupils with additional practice material? ________

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades? ________

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

UNIT VIII

Summary of Teachers' Comments

Reports were submitted by 20 teachers. The unit was taught to pupils at all levels but relatively few were described as low level. The number of teaching days varied from 8 to 26.

The following table shows the topics which were reported easiest to teach and the topics which were reported most difficult to teach. The number indicates how many teachers reported these topics.

Topic	Easiest to Teach	Most difficult to Teach
Transversal	4	0
Informal proofs	0	5
Principle 3	0	2
Angles of a triangle	3	0
Principles of parallel lines	1	0
Experiment	0	3
Applying principles	1	0
Converses	2	0
Isosceles triangle	2	0

It was apparent that the teachers thought that the vocabulary was difficult at times. Some of them stated that there was too much discovery; for example: cutting out triangles and angles reminded the student of cutting out paper dolls.

One teacher remarked that this unit did not go as well as the early units in her seventh grade class. One stated that for her eighth grade class it was an excellent unit.

Several teachers suggested measuring angles by using a protractor. They proposed that the idea of approximate measurement be introduced at the same time.

It was suggested by several that principles stated in final form should be in the guide for teachers but not in the students' units.

UNIT IX

INFORMAL GEOMETRY II

(Congruent triangles, perpendicular bisectors, parallelograms, and the right triangle principle).

This unit is written as a part of a sequence of units on geometry. In this sequence Informal Geometry II is intended to follow the units on Non—Metric geometry and Informal Geometry I. If the teacher does not wish to give this much time to geometry and prefers to try this unit rather than some of the others, this could be done providing the students have had some experience with the measure of length, area, and angle and also some experience with parallel lines and transversals.

In most schools Informal Geometry II should not be taught before the eighth grade. The writing committee believes that it would be appropriate to teach some of the geometry units in the seventh grade and some in the eighth.

Congruent triangles

The idea of congruence should be illustrated with numerous objects such as two sheets of paper, two textbooks, two light bulbs, and two footballs. Then the pupils should be led to see the importance of knowing whether two objects are the same size and shape, especially in business and manufacturing.

In manufacturing an automobile or a radio, the company wishes to make repairs as easy as possible by producing standardized inter-changeable parts, such as nuts, bolts, and tubes, and these must be of the same size and shape. In quality control certain measurements of a part are taken, and if too many specimens deviate from the standard by more than a certain amount, called the tolerance, the engineer knows that something has gone wrong in the

manufacturing process. Similarly, in manufacturing a complicated machine like an airplane engine, it is necessary to fit together many parts which were made separately. In order that the machine should work properly, the parts must be made in certain sizes and shapes to within very narrow tolerances.

When attention is turned to the congruence of two triangles, an excellent opportunity is provided to emphasize how the mathematician approaches a problem by considering a simple and quite "ideal" situation first. It can also be pointed out that the solution of a problem in such an "ideal" situation often proves to be very useful in solving a similar problem in life situations.

Some prefer to think of the movement of one triangle in a plane so that it coincides with a second triangle to which it is congruent, rather than the cutting out process selected in the text. The choice of the cutting out process was made since it seemed likely to be more clear to students. It is entirely satisfactory in an informal or intuitive treatment of geometry. It is, however, well to point out that solid figures in 3 dimensions could not be compared by similar process. In the text, we suggest a procedure of using molds for comparing solid figures.

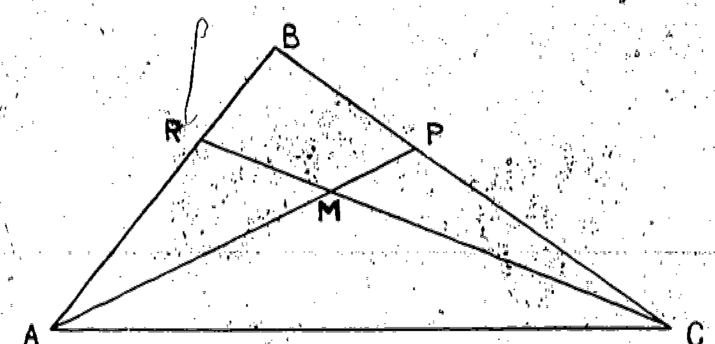
The concept of a rigid motion should be emphasized. Euclidean geometry can be defined as the study of those properties of figures which are left unchanged by rigid motions. On pages 2-4 we try to fix this idea in the minds of the pupils.

We try to bring out that we can see whether two figures are congruent, without physically moving them, by imagining how they would fit if they were moved. If the figure which is moved is cut

out of tracing paper, then corresponding points, i.e. points which lie over each other, can be marked, and after the figures are separated again, you may ask whether any distances with the figures were changed during the motion. Then ask how the distances between corresponding points of the two figures compare. Then ask whether the converse is true, whether if the two figures can be matched point for point so that distances between corresponding points are equal, the figures will be congruent.

On page 3 we try to get the children to see that they can draw with a compass the figure formed by all points in a plane whose distance from a given point is 4 inches. Thus they should discover that there are two possible positions for the point B, one on each side of the segment AC.

On page 4 the children may join the point M to the vertices A and C and extend the lines AM and CM until they meet the opposite sides in points P and R respectively:

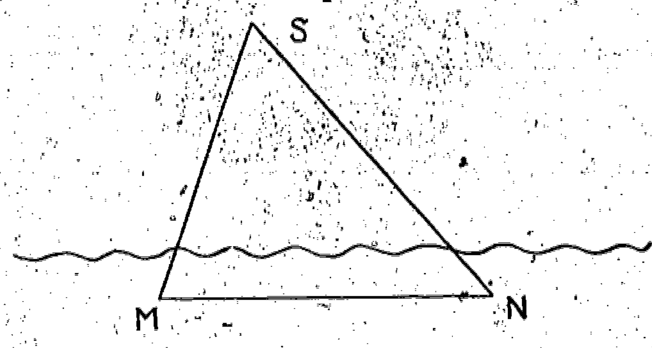


The points P' and R' may be located on C' B' and A' B' by measuring the distances from C to P and A to R and making the corresponding lengths equal. Then A' P' and C' R' intersect at the point M'.

As students begin to look for "tests" for congruence of two triangles, the teacher should emphasize the advantage of finding tests as simple as possible, that is tests involving the comparison

of as few parts of the two triangles as possible. Since it is assumed that pupils studying this unit will be familiar with the fact that the sum of the measures of the angles of a triangle is 180 degrees, it is probably a good idea to start with the case of two given angles and a side, even though this case may actually be more difficult for the pupils to see than some of the other cases.

On page 5 a practical application of tests for congruence of triangles is suggested. If the measurement of certain parts of two triangles is sufficient to show that they are congruent, then the measures of these parts determine uniquely the measures of all other parts. Another practical application is the determination of inaccessible distances. Suppose a man on shore wishes to find how far he is from a certain ship:



He may pick another nearby location N on the shore. Then with a sextant, he can measure the angles SMN and SNM and the distance MN. He can then make a drawing to scale and measure the distance SM. Later, when he studies trigonometry, he will learn how to calculate this distance.

On page 5, the distances of the airplane to Honolulu is 1710 miles, to three significant figures. In the scale drawing the length of HL is

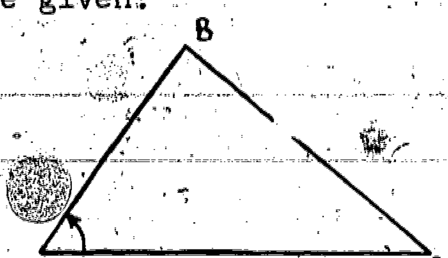
$$\frac{2620}{100} \cdot \frac{1}{8} = 3.28 \text{ inches}$$

or between 3 1/4 and 3 5/16 inches.

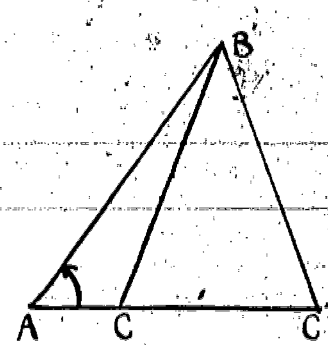
For the most part in this unit it is assumed that figures will be drawn with ruler and protractor. The teacher can use his own judgment about requiring construction with ruler and compass. The exercises reveal that in some situations there is actually an advantage in the use of compasses. It would be well for the teacher to use the conventional terms in connection with these instruments. When the figure is to be made by measuring with a ruler and protractor, it is conventional to use the phrase "draw the figure." When the compass is to be used it is conventional to say "construct the figure."

The exercises are intended to provide the opportunity for the student to explore the situations and discover the principles upon which there can be agreement, rather than to have these principles pointed out by the teacher and then verified by the pupil. This process may take longer but it would be better to provide for this and not cover so much material than to cover a great deal more material by teacher lecturing.

On page 10, special attention should be called to the case in which two sides and an angle (not included angle) are given. Let us suppose that sides AB and BC and the angle with vertex at A are given.



(1)



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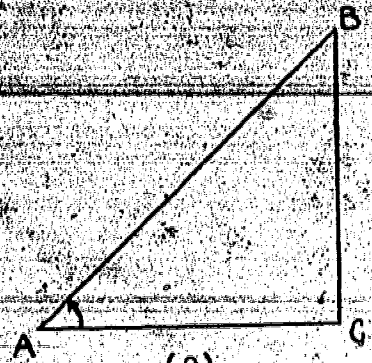
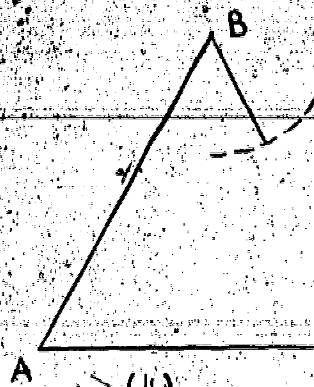




In (1), where BC is longer than AB, only one triangle can be drawn.

In (2), two different triangles can be drawn.

In (3), BC is just long enough to be perpendicular to side AC.

In (4), BC is too short to reach.

In trigonometry this case of given parts of a triangle is called the ambiguous case.

Special emphasis should be placed on the list of possibilities at the top of page 10, again emphasizing the advantage of the "discover" approach, rather than the memorizing of conclusions of others.

It should be made entirely clear that we have depended on intuition in developing the four principles on congruence. They are discovered in this unit by experiment and observation. The use of the word "prove" should be avoided throughout since we have not developed in the units on intuitive geometry a logical structure on which proof can be based. We might, however, call what we do in the parts following the treatment of congruent triangles, an "informal deduction process."

Perpendicular bisectors

As far as the chain of reasoning is concerned any one or all of the sections on perpendicular bisectors, parallelograms, or concurrent lines may be omitted. If the section on concurrent lines is studied, it would be desirable to precede it by the section on perpendicular bisectors. Otherwise any of the remaining sections may be studied in any order.

These three sections on perpendicular bisectors, parallelograms, and concurrent lines provide a nice opportunity to show certain properties of triangles and quadrilaterals using the informal deductive process in which most of the conclusions are based on the principles accepted for congruent triangles. These topics also provide opportunity for further discovery on the part of the pupils of properties not included in the text. The better students should be encouraged to try this.

In this section, as well as in those which follow, great stress should be laid on the discovery or verification of properties held by making use of previously accepted principles in the chain of reasoning. The ability to identify the reasons for conclusions, and steps used in reaching a given conclusion is more important than any one of the conclusions itself.

Parallelograms

The comments on perpendicular bisectors apply equally well in this section. Consideration of special figures like squares, rectangles, and rhombuses will provide a rich opportunity for discovery on the part of the better pupil.

The teacher may use the word "converse" if he chooses. Likewise this material may be adjusted for different classes or different pupils by stressing, or omitting altogether, the converse statements. One precaution should be mentioned. Great care should be exercised in the class to see that no conclusions are based on the converse of a statement, when the statement has been "shown" but not the converse. This is a common error in everyday reasoning. Students in the junior high school can profitably observe great care to avoid this.

On page 18, exercise 5c, it may help to draw with colored chalk the extensions of the segments HI, JK, and JH. If the children still do not see that JH is a transversal intersecting two parallel lines HI and JK, erase everything in the figure except these three lines, and ask again. If the pupils still don't see it, ask them to skim through Informal Geometry I again, and to try to find a figure similar to this one. If necessary, you should then ask them to read over the statement of principles for that unit again to see whether any of them applies to this figure and implies that two angles in the figure are equal.

Concurrent lines

Some of the most interesting work in elementary geometry is concerned with the concurrency of lines or line segments associated with a triangle. Pupils like to work with the drawings or constructions. This material also lends itself nicely to the discovery approach, and pupils, in working with this material, come to sense some of the orderliness and beauty of geometric relationships. Again in this section, much use should be made of measurement, and in the case of the perpendicular bisectors use should be made of the underlying argument to support the conclusion.

If the teacher wishes to spend more time on this material the pupils can be encouraged to experiment with the bisectors of the angles and with the medians, neither of which are mentioned in the text. The concurrency of the bisectors of two of the exterior angles of a triangle and the bisector of the other (interior) angle of the triangle would provide for further study of this idea on the part of pupils who may find it so interesting that they wish to give further time to the topic.

A Right Triangle Principle

Most eighth grade texts include work on the Theorem of Pythagoras and it is intended that the teacher use his own judgment in adding to the materials of this unit from what he may want to teach from his own text. This comment applies particularly to the applications of this principle to which no attention is given here. In other words this topic is treated more from the point of view of mathematical interest than of practical interest.

While the steps used in reaching the principle can in no sense be considered a proof, they do suggest a way of leading up to the principle by relating the new experience to previously learned principles which have been supported by informal deduction from other principles or which have been accepted on the bases of experimental evidence.

The teacher should keep clearly in mind that the purpose of the comparison of the two squares is to arrive at the conclusion that

$$r^2 + s^2 = t^2$$

where r and s are the lengths of the shorter sides of a right triangle and t is the length of the hypotenuse. An appeal is made to use of areas of squares to do this, which incidentally suggests why we might read r^2 "r-squared". The unit is written so that the student will arrive at this conclusion through the exercises rather than having it pointed out to him by the teacher and then proved.

The order of steps in determining Figure 2 is important. The pupil should first draw the square of 7 units on a side, and then in each corner measure off the triangles as suggested in the text. It then remains to argue that the quadrilateral, EFGH, is a square. This quadrilateral, of course, represents what we usually call the "square on the hypotenuse."

In writing this unit we have tried to introduce carefully and gradually the idea of expressing the result in the form of an equation:

At this point it is not crucial that the students master the idea of using letters to stand for numbers.

All right triangles, such as those on page 25, with integers as measures of the sides can be obtained from the formulas

$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2,$$

where m and n are arbitrary integers such that $m > n$. (Of course, a and b may be interchanged.) For example, if $m = 5$ and $n = 4$, we obtain case f on page 25, $a = 9$, $b = 40$, and $c = 41$.

The Babylonian tablet referred to on the previous page is Plimpton 322 in the Columbia University collection, and appears on plate 25 in Neugebauer and Sachs, Mathematical Cuneiform Texts, American Oriental Society, New Haven, Connecticut.

Pythagoras was born on the island of Samos about 572 B.C. In 532 he migrated to southern Italy and founded a semireligious brotherhood at Croton. This society became mixed up in politics, which led to attacks by enemies. Pythagoras then moved to Metapontium where he died about 495 B.C. You can obtain more information about his work from Heath, A History of Greek Mathematics, Oxford University Press. Plutarch's Lives is a very famous classic.

It is also assumed that the pupil will not have had much experience with the squares of numbers and the square roots of numbers. The use of the symbol, $\sqrt{\ }$, for square root has been avoided usually. The teacher should feel perfectly free to give a more adequate treatment if the pupils are prepared for it -- that is if they are already familiar with the ideas of square and square roots and the use of the symbol for square root.

In order to avoid other complications while the emphasis of attention is on geometric ideas, it is suggested in the unit that a table of square roots should be used. While it is not the place here to enter into the argument about the most appropriate method to teach the square root process, it does seem clear that all students should have had experience with reading a table of square roots.

The approximate nature of the square root of 5, for example, as read from a table or computed, cannot be emphasized too much.

If this is the first experience of the pupil with square root he should be given some exercises in which he finds such products as 1.4×1.4, 1.41×1.41, and 1.414×1.414. He should see that while none of these numbers 1.4, 1.41, and 1.414 is a square root of 2, that each of these gives an approximation of the square root of 2, which may be satisfactory for some purposes. It would be well also to compare the product 1.41×1.41, which is less than 2, with the product 1.42×1.42, which is more than 2. This comparison shows that the square root of 2 lies somewhere between 1.41 and 1.42.

The pupil should also recognize that the "square root of 2" is a number even though he may not be able to express it exactly using decimals or fractions.

Exercises 11 and 12 are intended to show that we can by geometric considerations [illegible] the length of which is the square root of 2. These problems are starred and may, of course, be omitted if the teacher chooses.

(Answers to the exercises)

Part A. Congruent triangles.

	A	B	C	D	L
A	0	4	6		
B		0	5		
C			0		
D					
L					

The measures of the angles A, B, and C are, respectively,

$$A \overset{m}{=} 55\ 46'\ 1[illegible]'', \qquad B \overset{m}{=} 82\ 29'\ 9'', \qquad C \overset{m}{=} 41\ 24'\ 35'',$$

and

$$\cos A = 9/16 = .5625, \quad \cos B = 1/8 = .125, \quad \cos C = 3/4 = .75$$

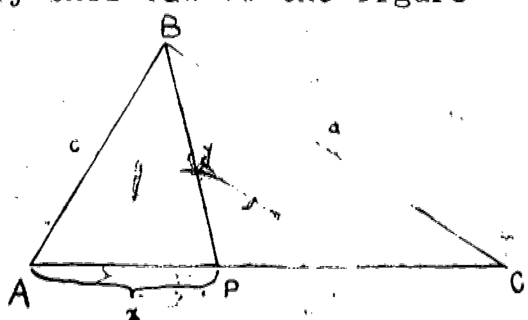
The various distances may be computed by the law of cosines: if a, b, and c are the lengths of the sides of the triangle ABC opposite the angle A, B, and C, respectively, then

$$a^2 = b^2 + c^2 - 2bc \cos A, \text{ etc}$$

In order to calculate the measure of A, we have solved for cos A:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36 + 16 - 25}{2 \cdot 6 \cdot 4} = \frac{27}{48} = \frac{9}{16}.$$

If we wish to calculate the distance from B to D, E, F, G, and H, then we may apply this law to the figure



and obtain the relation

$$y^2 = c^2 + x^2 - 2c\,x\cos A$$
$$= 16 + x^2 - \frac{9}{x},$$

which yields the table

x	y		
1	$\sqrt{12.5}$	=	3.534
2	$\sqrt{11}$	=	3.317
3	$\sqrt{11.5}$	=	3.391
4	$\sqrt{14}$	=	3.742
5	$\sqrt{18.5}$	=	4.301

The distances in the rest of the table may be computed in the same way.

Of course, the children are supposed to obtain these distances by measurement, instead of computation. We have included here this additional information for your benefit. There may be an occasional pupil who would profit from the discussion here.

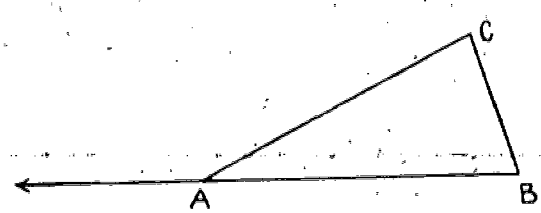
	A	B	C	D	E	F	G	H	J	K	L
A	0	4	6	1	2	3	4	5	1	2	3
B	4	0	5	3.53	3.32	3.39	3.74	4.30	3	2	1
C	6	5	0	5	4	3	2	1	5.50	5.15	4.98
D	1	3.53	5	0	1	2	3	4	.94	1.66	2.57
E	2	3.32	4	1	0	1	2	3	1.66	1.87	2.50
F	3	3.39	3	2	1	0	1	2	2.57	2.50	2.81
G	4	3.74	2	3	2	1	0	1	3.67	3.32	3.39
H	5	4.30	1	4	3	2	1	0	4.51	4.21	4.14
J	1	3	5.50	.94	1.66	2.57	3.67	4.51	0	1	2
K	2	2	5.15	1.66	1.87	2.50	3.32	4.21	1	0	1
L	3	1	4.98	2.57	2.50	2.81	3.39	4.14	2	1	0

You may ask the children to fill in the first row, and then to find an easy way to fill in the first column. Try to see how long it takes the pupils to discover the symmetry with respect to the main diagonal. Let the children compare the triangles JAD, KAE, CAF, and BAG, the triangles JAE, KAG, and LAC, the triangles JAF and KAC, and such pairs as triangles JAF and KAE. They may notice that these families of triangles are similar, and that the lengths of corresponding sides are proportional.

1. The triangles are congruent
2. 60°
3. The triangles are congruent, but not congruent to the triangle in Ex. 1.
4. The triangle is not congruent to those of numbers 1 and 3.
5. If 2 angles and the included side of one triangle are equal in measure to 2 angles and the included side of another triangle, the triangles are congruent.
6. b and c, a and d
7. a, c, and e, d and f
8. No. Sides included by the angles with the same measure do not have the same measure.
9. The triangles are not necessarily congruent.

Part B: EXERCISES IN EXPLORING TRIANGLES

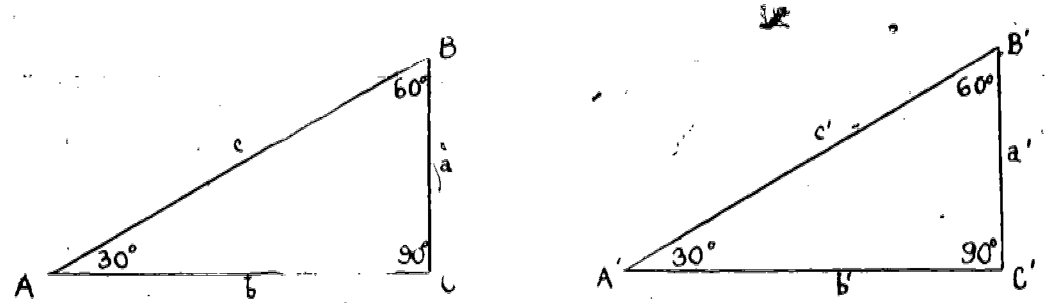
1. Yes
2. Yes
3. The triangles appear to be congruent
4. The triangles appear to be congruent
5. True on the basis of the triangles compared
6. All 6 triangles are congruent. All sides are equal. All the triangles are equilateral since all their angles have a measure of 60 degrees.
7. (a) Draw a line segment equal to the measure of one side, call it AB. Place a thumb tack at A and another at B. Tie knots in the string dividing the string into lengths that have the same measures as the second and third sides of the triangle. Attach the end knots to the thumb tacks at A and B. Stretch the string tight and place a thumb tack at the third knot C to form triangle ABC.



Part C: CONDITIONS FOR CONGRUENCE OF TRIANGLES

Measurement of the angles of two triangles is not sufficient

to tell whether the triangles are congruent. If the corresponding angles of two triangles are equal in measure (case I), then the triangles are similar. The quotients of the lengths of corresponding sides in the two triangles are equal. For example, in the triangles below,



if a, b, c, a', b', and c' are the lengths of the sides, then

$$\frac{a}{a'} \quad \frac{b}{b'} \quad \frac{c}{c'}.$$

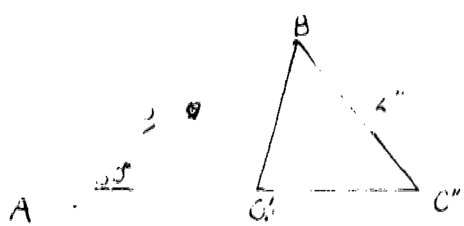
The sum of the measures of the angles of a triangle is 180 degrees. Therefore, in case V, if two angles of one triangle are equal in measure to the corresponding angles of another triangle, then so are the third angles of the triangles equal in measure. Thus case V can be reduced to case III.

1. Triangles are congruent. There is only one way to draw the triangle under these conditions.

2. The third angle will have the same measure

   angle C $\stackrel{m}{=}$ 95°, angle F $\stackrel{m}{=}$ 55°, the triangles are congruent.

3.



The possible positions of the point C on a given side of the line AB such that BAC = 30 degrees form a ray with endpoint at A. The set of all points C such that the distance from B to C is 2 inches is the circle with center at B and radius 2 inches. The intersection of these two sets consists of 2 points, indicated by C' and C" above. The possible lengths of AC are

$$AC = \frac{3\sqrt{3} - \sqrt{7}}{2} = 1.28", \quad AC = \frac{3\sqrt{3} + \sqrt{7}}{2} = 3.92".$$

In case V, if two angles and a side opposite one of them in one triangle are equal in measure to the corresponding parts of another triangle, then the triangles are congruent.

In case VI, if two sides and an angle opposite one of them in one triangle are equal in measure to the corresponding parts of another triangle, then the triangles are not necessarily congruent. See ex [illegible] above.

Part D: [illegible] TRIANGLES

1. (a) [illegible] (b) [illegible] ( ) [illegible] ( ) [illegible] (e) II (f) [illegible]

   (g) II (h) [illegible]

2. c and d (h) [illegible] principle 10 of the previous chapter)

Part E: [illegible]

1. (a) by const[illegible]
   (b) each one [illegible]
   (c) principle 4
   (d) principle 1

2. No

3. No

4.

5. (e) [illegible] $\overset{m}{=}$ [illegible] yes

6. AO $\overset{m}{=}$ BO. [illegible] $\angle$ AOP $\overset{m}{=}$ $\angle$ AOP +

   $\angle$ AOP $\overset{m}{=}$ $\angle$ BOP $\overset{m}{=}$

8. (c) yes

9. A[illegible] AS, by construction
T[illegible]gles congruent by Principle 3.
A[illegible] ABR and ABS are 90°, since they are equal in measure
a[illegible]e sum of their measures is 180°.
Th[illegible] is the perpendicular bisector at RS.

10. No

12. One is the converse of the other

13. E and F lie on the perpendicular bisector at CO, by principle 6. There is no other line on E and F except ℓ, thus ℓ is the perpendicular bisector of CD.

Part F: PARALLELOGRAMS

1. b and d

2. None

4. The triangles are congruent. AB $\overset{m}{=}$ DC, BC $\overset{m}{=}$ AD, $\angle B \overset{m}{=} \angle D$, $\angle A \overset{m}{=} \angle C$.

5. a. Principles 1 and 5 in this chapter
   b. Principle 1 if HJ is drawn
   c. HI and JK are parallel, intersected by the transversal HJ. IHJ $\overset{m}{=}$ HJK, since they are alternate interior angles (principle 7 in the previous chapter.) Similarly, HK and IJ are parallel lines intersected by the transversal HJ, and HJI $\overset{m}{=}$ KHJ for the same reason.
   d. Triangle HJK is congruent to triangle JHI, by Principle 2.

6. a. Definition of a parallelogram.
   b. Principle 7 of the previous chapter
   c. Same reason
   d. Any number is equal to itself.
   e. Principle 2
   f. Principle 1

8. AB $\overset{m}{=}$ CD. Alternate interior angles at A and C, B and D are equal in measure. AM $\overset{m}{=}$ MC and DM $\overset{m}{=}$ BM since triangles AMB and CMD are congruent by principle 2.

9. The sum of the measures of $\angle$ A and $\angle$ B is 180 degrees. If AB is extended, then it is seen to be a transversal intersecting the lines BC and AD in such a way that corresponding angles have equal measures. Therefore BC and AD are parallel. Similarly AB and CD are parallel, and so ABCD is a parallelogram, by definition.

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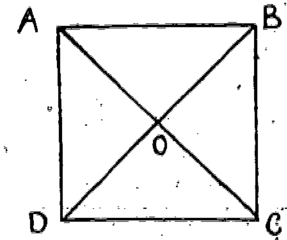
10. a. Triangles ABC and CDA are congruent.
    b. Principle 3
    c. $\angle ADB \overset{m}{=} \angle CBD$ and $\angle ABD \overset{m}{=} \angle CDB$.
    d. Principle 4 of the previous chapter.

11. The quadrilateral is a parallelogram.
    If the diagonals at a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



12. The statement in number 8 is probably true.

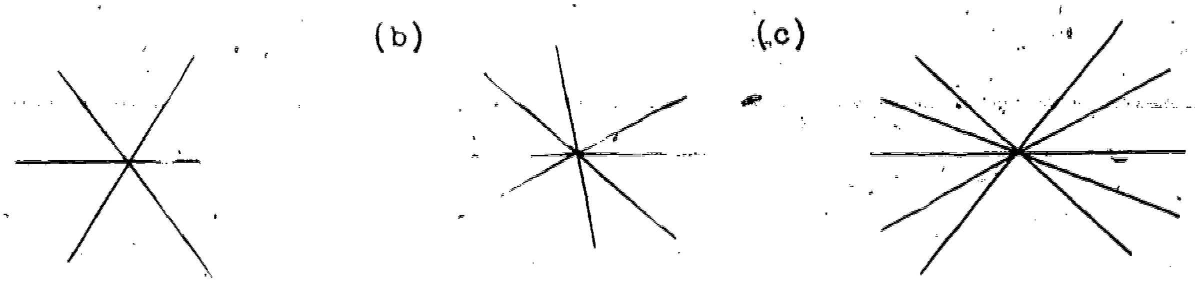
13. Show that triangles ABO and BCO are congruent. Then angle AOB $\overset{m}{=}$ angle BOC $\overset{m}{=}$ 90°

14. False

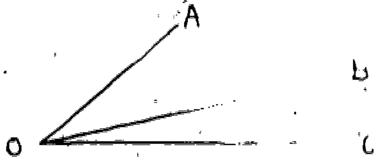
15. Example: the diagonals of a rectangle are equal in length.

Part G: CONCURRENT LINES

1. (a) (b) (c)



2.



3. Three, AOB, BOC, and AOC

4. a, c and e

5. The perpendicular bisectors should be concurrent

6. The perpendicular bisectors are concurrent in any triangle

9. The altitudes are concurrent

Part H: COMPARISON OF SQUARES

1. a. 49 square inches
   b. 9 and 16 square inches
   c. 12 square inches.

3. AF = BG, AE = BF, angle A = angle B. Principle 4.

4. Principle 1

5. AEF and BFG, 180°, 90°, 180°, 90°, 90°

6. Square

7. Yes. Principle 4.

8. The measures of the area at square EFGH of figure 2 and the sum of the measures of the areas of the squares in figure 1 are the same.

9. The area of the square with length of sides EF is measured by the sum of the area of the squares of the two shorter sides of the triangle AFE.

10. Yes. EF = 13 inches

Part I: A RIGHT TRIANGLE PRINCIPLE

	a	b	c	m	n
a.	5	12	13	3	2
b.	7	24	25	4	3
c.	8	15	17	4	1
d.	20	21	29	5	2
e.	12	35	37	6	1
f.	9	40	41	5	4
g.	28	45	53	7	2
h.	11	60	61	6	5
i.	56	33	65	7	4
j.	16	63	65	8	1
k.	48	55	73	8	3
l.	13	84	85	7	6
m.	36	77	85	9	2
n.	39	80	89	8	5
o.	65	72	97	9	4
p.	20	99	101	10	1

This table was constructed by using the fact that if m and n are any two integers with $m > n$, then $m^2 - n^2$, $2\,mn$ and $m^2 + n^2$ are the lengths of the sides of a right triangle. You may verify the identity

$$(m^2 - n^2)^2 + (2\,mn)^2 = (m^2 + n^2)^2.$$

In some of the above cases $a = m^2 - n^2$ and $b = 2\,mn$, and in others $a = 2\,mn$ and $b = m^2 - n^2$. It is an interesting theorem that all right triangles, the lengths of whose sides are integers, can be obtained in this way.

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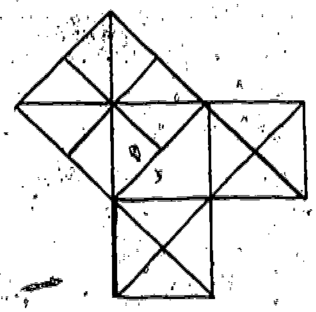
Part J: EXERCISES ON THE PYTHAGOREAN THEOREM

1.



2. The short sides of the right triangle have the same measure.

3. a. $25 = 16 + 9$ b. $169 = 25 + 144$
 c. $625 = 49 + 576$ d. $400 = 256 + 144$

5. a. 2.2 b. 6.4 c. 3.6

6. a. 5 b. 41 c. 13

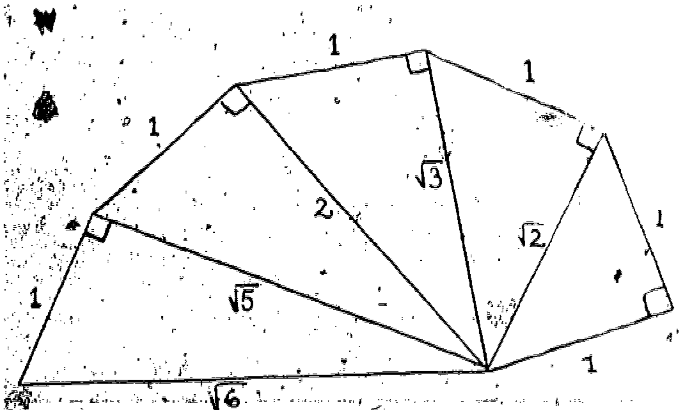
7. a. 2.24 b. 6.40 c. 3.61

8. If you know the measure of the area of a square, you find its side by taking the square root of this measure. Measured values of square roots should be approximately the same as completed value.

9. a. 5.8 b. 7.8 c. 14.9 d. 32

10. Square root of 2 = 1.4" 11. $\sqrt{3} = 1.7"$

12. a. 2.45 b. 2.65



UNIT IX

Sample Test Questions

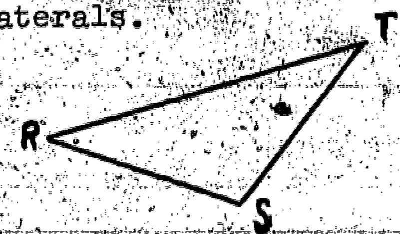
PART I. TRUE - FALSE.

F 1. All isosceles triangles are congruent.

F 2. Pythagoras was the first man to discover the Pythagorean Theorem.

T 3. All parallelograms are quadrilaterals.

T 4. In the triangle at the right, the side RT is included between the angles SRT and RTS.



F 5. Three or more li[illegible] a point are said to be concentric.

T 6. The hypotenuse of a right triangle is always the longest side of the triangle.

F 7. All quadrilaterals are parallelograms.

F 8. In the four sided figure ABCD shown at the right, if sides AB and CD are parallel, and if sides $AD \overset{m}{=} BC$, the resulting figure will always be a parallelogram.

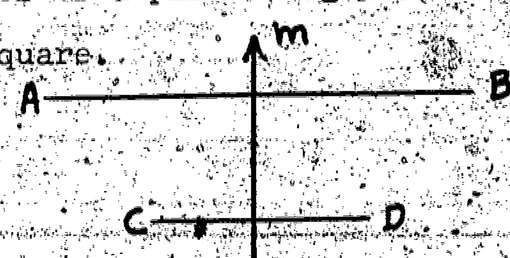
T 9. A triangle may never include two right angles.

F 10. If the measures of the four sides of a parallelogram are equal, the figure is always a rectangle.

F 11. The altitudes of a triangle are concurrent.

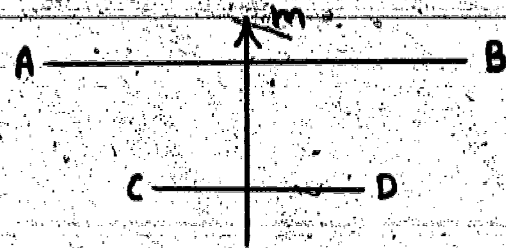
F 12. If the measures of all the angles of a parallelogram are equal, the figure is always a square.

F 13. In the figure at the right, lines AB and CD are parallel. If line M is the perpendicular

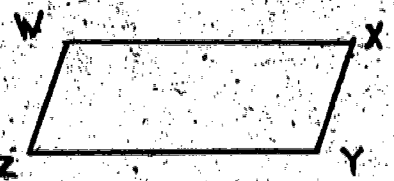


bisector of AB, it also bisects CD.

T 14. In the figure at the right, if line M is the perpendicular bisector of lines AB and CD, lines AB and CD are parallel.

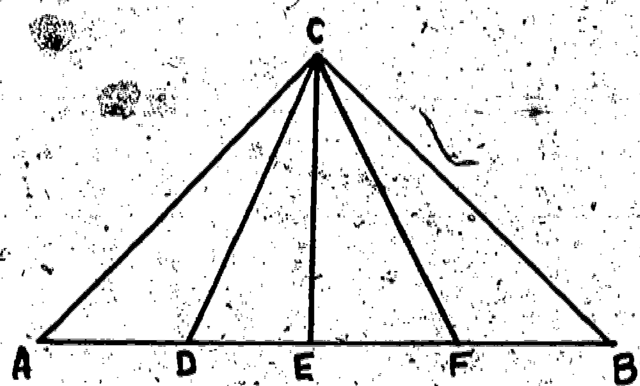


F 15. In the figure at the right, if sides WX and YZ are parallel, WXYZ is a parallelogram.



Items 16 through 20 are based on the figure shown at the right in which AC $\overset{m}{=}$ BC and AD $\overset{m}{=}$ BF.

Mark the following statements true or false on the basis of the information given above.



F 16. DE $\overset{m}{=}$ EF.

T 17. CD $\overset{m}{=}$ CF.

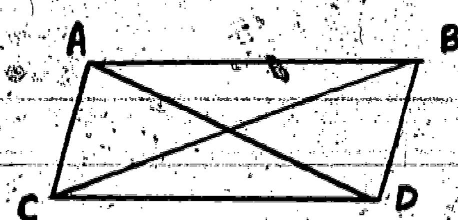
F 18. AE $\overset{m}{=}$ BE.

F 19. Angle CED $\overset{m}{=}$ angle CEF.

T 20. Angle CDE $\overset{m}{=}$ angle CFE.

PART II. MULTIPLE CHOICE.

1. The figure at the right is a parallelogram. Which of the following statements <u>cannot</u> be proved?



A. Triangles ABD and ACD are congruent.

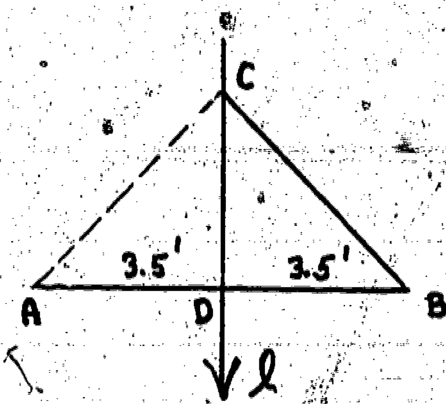
*B. AO $\overset{m}{=}$ OC.

C. Angle BOD $\overset{m}{=}$ angle AOC.

D. Angle BAD $\overset{m}{=}$ angle ADC.

E. Triangles AOC and BOD are congruent.

2. In the figure at the right, line ℓ intersects AB at D so that AD $\overset{m}{=}$ BD and both lines measure 3.5 feet. Angle ADC $\overset{m}{=}$ angle BDC. If BC $\overset{m}{=}$ 8.5 feet, the measure of AC is equal to the measure of



A. 5.0 feet.

B. AD.

C. DC.

D. 3.5 feet.

*E. 8.5 feet.

3. How many concurrent lines may there be on a given point?

A. 0

B. 1

C. 2

D. 3

*E. Any number.

4. How many figures would you need to draw to prove that the altitudes of a triangle are concurrent.

A. 1

B. 2

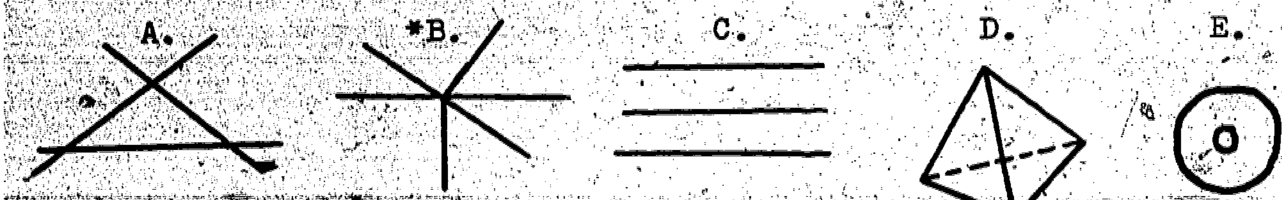
C. 3

D. 10

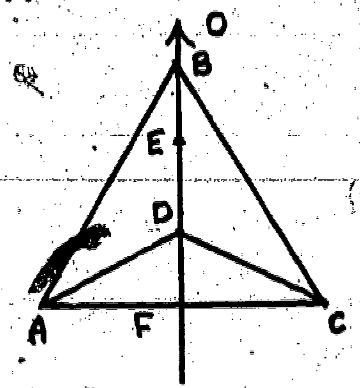
*E. You cannot prove a statement by making drawings.

5. In which one of the following does it appear that <u>all</u> the

lines or rays in the figure are concurrent.



6. In the figure shown at the right AB $\stackrel{m}{=}$ BC and AD $\stackrel{m}{=}$ DC. If BD is extended to meet AC on F, then...



*A. AE $\stackrel{m}{=}$ CE.

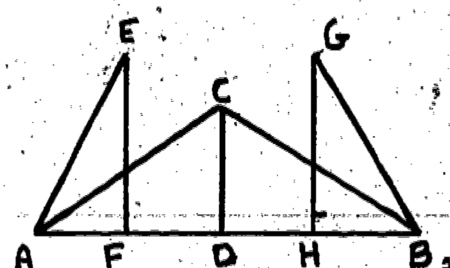
B. BE $\stackrel{m}{=}$ DE.

C. ~~AD $\stackrel{m}{=}$ BD.~~

D. AC $\stackrel{m}{=}$ AB.

E. CD $\stackrel{m}{=}$ EF.

7. In the figure at the right, if CD is the perpendicular bisector of AB, and lines EF, CD, and GH are parallel, then...



A. AE $\stackrel{m}{=}$ BG.

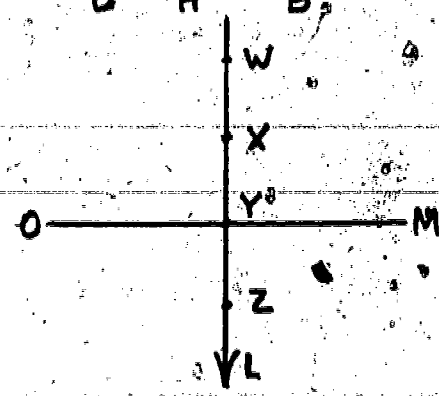
*B. AE $\stackrel{m}{=}$ AC.

*C. AC $\stackrel{m}{=}$ BC.

D. BC $\stackrel{m}{=}$ BG.

E. None of the above is correct.

8. In the figure at the right, line L is the perpendicular bisector of OM. Points W, X, Y, and Z represent possible locations of a buried treasure. If the only clue to the location of the treasure is that it is buried at at a point which measures an equal distance from points O and M,



then the treasure will be at point...

A. W.

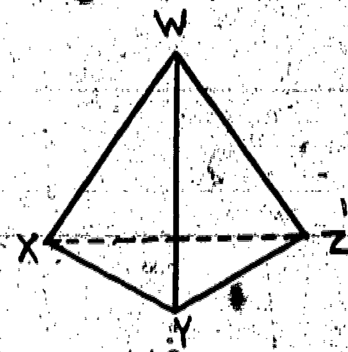
B. X.

C. Y.

D. Z.

*E. Cannot tell from the above information.

9. In the triangular pyramid at the right the base XY Z is an equilateral triangle and the faces (sides) WXY, WYZ, and WXZ are congruent, isosceles triangles.



Which of the following statements is not correct?

A. $WZ \cong WY$.

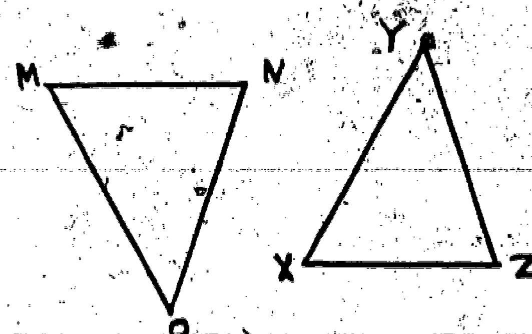
B. $WX \cong YW$.

C. $XZ \cong YZ$.

D. $XW \cong ZW$.

*E. $XY \cong WZ$.

10. In the pair of triangles at the right, angle OMN $\cong$ angle YZX, and line MN $\cong$ line YZ. What is the least amount of additional information necessary to be able to prove that triangle MNO is congruent to triangle XYZ?



A. $NO \cong XY$.

*B. $MO \cong XZ$.

C. Angle NOM $\cong$ angle YXZ.

D. $NO \cong XY$ and $MO \cong XZ$.

E. None of the above will prove the triangles congruent.

11. How many elements are in the intersection set of the perpendicular bisectors of the sides of a triangle?

A. 0

*B. 1

C. 2

D. 3

E. Any of the above may be correct.

12. Which of these triangles are right triangles according to the length of the sides given?

A. [illegible] II

B. II and III

C. III and IV

*D. I and III

E. All of these triangles are right triangles.

13. Which one of the following formulas is used to represent the Pythagorean Theorem?

A. $\frac{A^2}{B^2} = C^2$

*B. $n^2 + o^2 = z^2$

C. $a + b = c^2$

D. $(a + b)^2 = c^2$

E. Answers B and C are both correct.

14. Which of the following sets of three numbers are possible measures of the sides of right triangles?

I.	II.	III.	IV.
9, 12, [illegible]	12, 5, 13	4, 9, 10	7, 24, 25

A. Only I and II

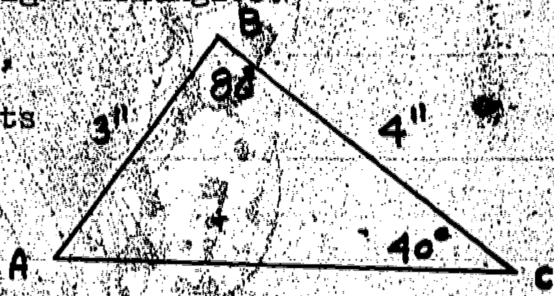
B. Only II and III

C. Only I, II, and IV

D. Only I, II, and III

E. All of these are possible right triangles.

15. Using the triangle at the right, which of the following statements are true?



I. AC $\stackrel{m}{=}$ 5 inches

II. The area of triangle ABC $\stackrel{m}{=}$ 6 square inches.

III. AC must be less than 7 inches.

IV. AC must be the longest side.

A. Only I and II are true.

*B. Only III and IV are true.

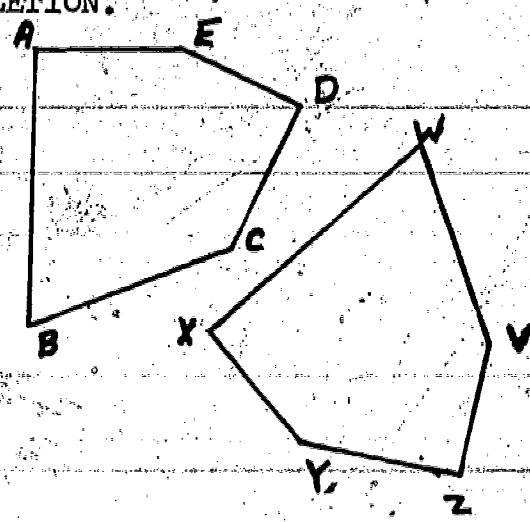
C. Only I and III are true.

D. Only I, III, and IV are true.

E. I, II, III, and IV are all true.

PART III. COMPLETION.

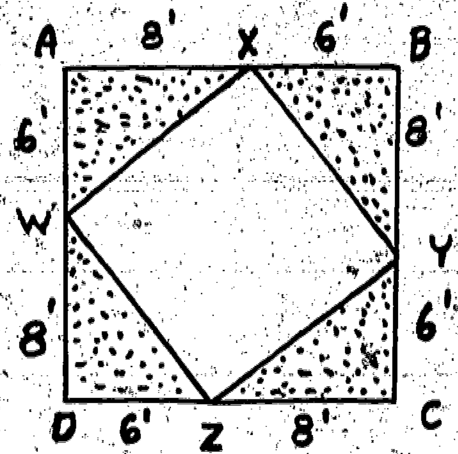
Items 1 and 2 -- The two figures at the right are congruent with angle EAB $\stackrel{m}{=}$ angle WXY, and angle ABC $\stackrel{m}{=}$ VWX. Write down the line segment that corresponds to each of the following segments.



1. AB and (WX or XW)

2. DC and (VZ or ZV)

Items 3 through 6 --- Use the figure on the right to answer the items below.



3. What is the area of the large square ABCD? (196 sq. ft.)

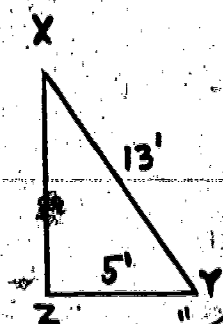
4. What is the area of the small square WXYZ? (100 sq. ft.)

5. What is the measure of the length of XW? (10 ft.)

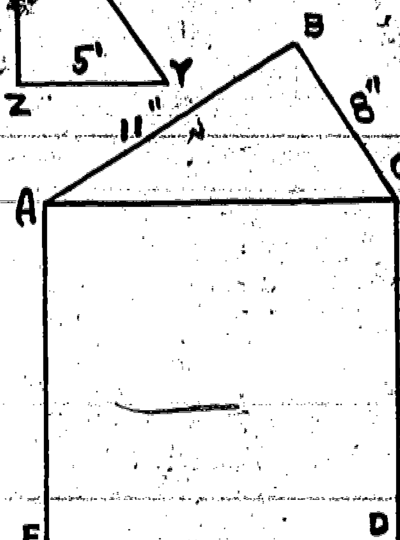
6. What is the measure of the sum of angles AXW and AWX? (90°)

7. If you can place one square on top of another square so that the four vertices of the first square lie exactly on the four vertices of the second square, you can say that the two squares are (congruent).

8. XZ is a telephone pole, XY the length of a wire support and YZ the distance along the ground from the pole to the place where the wire support is anchored to the ground. If XY $\stackrel{m}{=}$ 13 feet and YZ $\stackrel{m}{=}$ 5 feet, what is the height of the telephone pole? XZ $\stackrel{m}{=}$ (12 ft.)



9. In the right triangle ABC, what is the area of the square on AC if AB $\stackrel{m}{=}$ 11 inches and BC $\stackrel{m}{=}$ 8 inches. Area of ADEC $\stackrel{m}{=}$ (185 sq. in.)



PART IV.

Questions 1 to 5 apply to the figures <u>SHOWN BELOW</u>. Decide whether the triangles ABC and DEF will be congruent using <u>only</u> the information given in each question.

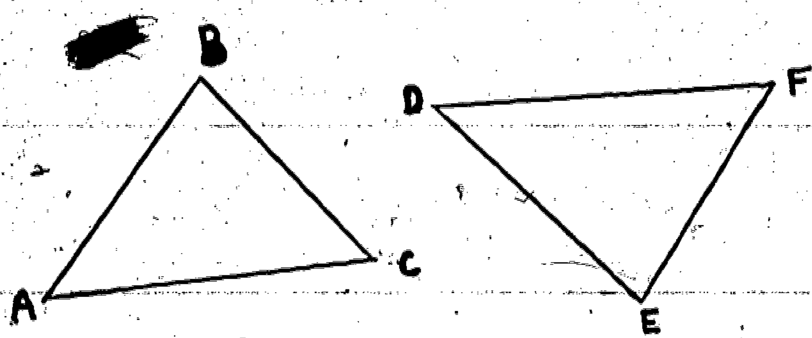
1. Angle A $\overset{m}{=}$ angle F.
   Angle C $\overset{m}{=}$ angle D.
   AC $\overset{m}{=}$ DF. <u>(Congruent)</u>

2. BC $\overset{m}{=}$ DE.
   AB $\overset{m}{=}$ FE.
   Angle A $\overset{m}{=}$ angle E. <u>(Not congruent)</u>

3. BC $\overset{m}{=}$ FE.
   DE $\overset{m}{=}$ AB.
   AC $\overset{m}{=}$ DF. <u>(Congruent)</u>

4. Angle A $\overset{m}{=}$ angle F.
   Angle B $\overset{m}{=}$ angle E.
   Angle C $\overset{m}{=}$ angle D. <u>(Not congruent)</u>

5. Angle B $\overset{m}{=}$ angle E.
   Angle C $\overset{m}{=}$ angle F.
   AC $\overset{m}{=}$ DF. <u>(Congruent)</u>



Date ________________

UNIT: ____

Name of Teacher: ____

Name of School: ____

City: ______________________ State: __________

Number of days given to the teaching (including testing) of this unit: ______________

Approximate dates: ______________________________________

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?

   Did you omit any part? ________

4. Did you use any supplementary developmental materials? ______

   If so, what were they, and at what points were they used?

5. Did you find it necessary to provide the pupils with additional practice material? ________

   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades? ________

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

Summary of Teachers' Comments

Four teachers reported on Unit IX. They gave the number of days spent on the unit as 13, 15, 18, 18, and described the classes as average except for one which was made up of gifted pupils.

The following topics were reported as easiest to teach:

- Congruent triangle
- Perpendicular bisectors
- Right triangles
- All of the unit
- Concurrent lines

The following topics were reported as most difficult to teach:

- Concurrency
- Theorem of Pythagoras
- Most of the unit
- Applications of congruency

Since only 3 teachers reported the comments are limited. All three seemed to think that it was a difficult unit. This was to be expected as the unit was intended to be for eighth grade pupils.

"Proofs" did not go well in any of the classes.

MEASUREMENT AND APPROXIMATION

The process of measurement plays such an important part in contemporary life that everyone should have a clear understanding of its nature. A substantial part of the arithmetic taught in the elementary school relates to measurement. Most of the early work in measurement is designed to familiarize children with common units of measure and their use, and with the ratios between them.

The basic concept to be developed in this unit is that the process of measurement of a single thing yields a number which represents the approximate number of units. This is in contrast to the process of counting separate objects, which yields an exact number. When the number of separate objects is rounded or estimated, the resulting number is treated as an approximation in the same sense as a measurement. Since measurements are approximate, calculations made with measurements, such as areas, yield results which are also approximate.

The unit selected for a given measurement should be suitable for the thing to be measured, and for the purpose for which the measurement is to be used. The unit is not necessarily a standard unit, but may be a multiple of a standard unit, or a fraction of a standard unit. For example, the

height of an airplane above the ground may be stated using 100 feet as a unit. The height of a person may be stated to the nearest half-inch, that is, using the half-inch as a unit. The result of measurement should be stated so as to indicate what unit was used. A measurement reported as 2 3/4 in. implies that the unit used was 1/4 inch, and that the result is stated to the nearest fourth-inch. The result, "2 3/4 in." thus applies to any measurement which is within half a unit on either side of the 2 3/4 inch mark; that is, which is more than 2 5/8 and less than 2 7/8 inches. We have used the term "greatest possible error" to refer to the amount by which the actual measurement may vary from the stated result. The use of the word "error" may cause some difficulty. It should be emphasized that the word is used here to mean that a measurement such as 2 3/4 inches represents any of a series of measurements ranging from 2 3/4 - 1/8 to 2 3/4 + 1/8. This meaning should be distinguished from the more familiar use of the term to mean mistakes in using the measuring instrument, mistakes in reading the scale, or mistakes resulting from use of a faulty instrument, such as a poorly marked ruler.

The two terms, "precision" and "accuracy" are sometimes confusing. In common usage, precision refers to the size of the unit of measurement used; the smaller the unit, the more

precise the measurement. This means, of course, that the more precise the measurement, the smaller is the greatest possible error implied. Accuracy of a measurement is indicated by the per cent the greatest possible error is of the measurement.

In order to develop clearly the concepts of unit of measurement and greatest possible error, it is suggested that the students be given considerable practice in measuring lines to the nearest inch, nearest half-inch, nearest tenth of an inch, and so on. They should be asked to state the greatest possible error for measurements, and to indicate within what range a stated measurement must fall.

The greatest possible error of the measurement of the area of a rectangle computed from linear measurements is shown first by means of a drawing and then by computation. The distributive law is then used to analyze the product of the two dimensions in order to determine the greatest possible error of the computed area. It would be well to relate the development by means of the distributive law to the drawing so that the areas represented by the various products (3 1/4 x 1/8, 1 3/4 x 1/8, and so on) can be identified in the drawing. This is especially suggested in case the students have not already become thoroughly familiar with the distributive law.

Students sometimes have difficulty in seeing why measurements with the same significant digits have the same accuracy, regardless of the unit of measure. A development along the following lines is sometimes helpful. Significant digits are defined as the digits in the numeral which show the number of units. The measurements below are analyzed to show the unit, the number of units, and the possible error.

3570 ft. Unit: 10 ft. No. of units: 357 Possible error: 5 ft.

.0357 ft. Unit: .0001 ft. No. of units: 357 Possible error: .00005 ft.

Each of these measurements contains three significant figures. To determine the accuracy, we find the relative error, or per cent of error. In the first case, we have the ratio $\frac{5}{3750}$

In the second case, the ratio is $\frac{.00005}{.0357}$, which equals $\frac{5}{3750}$

Students also have difficulty in understanding why measurements with a larger number of significant digits have greater accuracy. An example similar to that above can be used.

3.57 ft. Unit: .01 ft. No. of units: 357 Possible error: .005 ft.

35.70 ft. Unit: .01 ft. No. of units: 3570 Possible error: .005 ft.

Relative error of 3.57 ft. is $\frac{.005}{3.57}$, or $\frac{5}{35700}$

Relative error of 35.70 ft. is $\frac{.005}{35.70}$, or $\frac{5}{3570}$

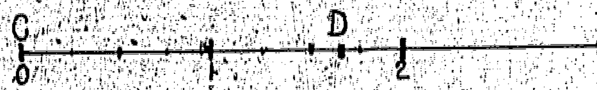
If it seems desirable, this unit might be increased in scope by reports on such topics as the history of standard units of measure, various measuring devices used in industry and science, and the work of the United States Bureau of Standards.

References

Butler, C. H. and Wren, F. L. Teaching of Secondary Mathematics, 279-289. New York: McGraw Hill Book Company, 1951.

ANSWERS

Exercises pp. 3-4

1. 

CD = $1\frac{3}{4}$

2. Between 1 5/8" and 1 7/8"; 1/8" from 1 3/4"
3. 1/8"
4. The point D may be anywhere between 1 5/8" (1 3/4 - 1/8) and 1 7/8" (1 3/4 + 1/8)
5. a) 1/8   b) 1 1/16" and 1 3/16"   c) 1/16"   d) 1/16
6. Between 2 9/32" and 2 11/32"
7. 1/16   1/16
8. one-half of the unit used
9. 1/10"; 1/20"; 3 7/10 ± 1/20
10. 1/200"

Exercises pp. 5-8

1. 3.20 inches
2. 4.0 inches
3. a) .1 ft. ; .05 ft. ; 1/4 ft. ; 1/8 ft. ; 5.2 is more precise
   b) .01 ft. ; .005 ft. ; .1 ft. ; .05 ft. ; .68 is more precise
   c) .001 in.; .0005 in.; .001 in.; .0005 in.; same precision
4. $12\frac{1}{2}$ and $13\frac{1}{2}$ years old
5. This depends on when the question is asked. (the present month plus or minus three months)
6. a) 100 ft.; 50 ft.   b) 1 ft.; .5 ft.   c) 10 ft.; 5 ft.
   d) .1 ft. ; .05 ft.   e) .0001 ft.; .00005 ft.
   f) .1 ft. ; .05 ft.
7. e is the most precise; a is the least precise; yes, d and f
8. a) 420<u>0</u>   b) 23,<u>0</u>00   c) 48,0<u>0</u>0,000

Exercises p. 9

1. a) .5 ft.   b) .05 in.   c) 5 mi   d) .5 ft.
   e) .005 in.   f) .0005 ft.   g) 500 mi   h) 5 mi
2. a) .96%   b) 1.2%   c) .19%   d) .14%
   e) .071%   f) 8.3%   g) .93%   h) .0093%
3. a) .05 ft; .54%   b) .0005 ft; .54%   c) 5 ft; .54%
   d) 500 ft; .54%

4. The relative errors are the same for each measurement. In each case, the greatest possible error was the same fractional part of the measurement.

Exercises pp. 10-11

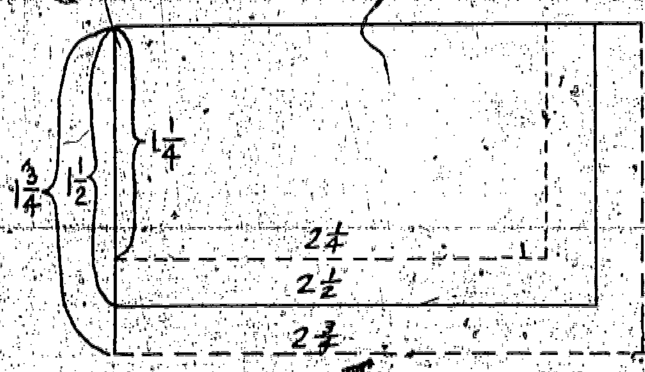
1. a) 52 x 10 ft; 52 b) 3,246 x .01 in; 3246 c) 2 x .001; 2
   d) 4,036 x .1 ft; 4036 e) 2,580 x 10 ft; 2580
   f) 15 x .0001 in; 15 g) 3,890 x .01 ft; 3890
   h) 603 x .0001 in; 603
2. a) .19% b) .19% c) .19%
   d) .98% e) .98% f) .98%
3. 1. a) (nearest) 10 ft. b) .01 in. c) .001 in. d) .1 ft.
      e) 10 ft. f) .0001 in. g) .01 ft. h) .0001 in.
   2. a) .1 ft. b) .001 ft. c) 10 ft. d) 1,000 mi
      e) .1 ft. f) .001 in.
4. a,b,&c have the same accuracy; d,e,&f have the same accuracy.
   a,b,&c have the same significant digits; d,e,&f have the same significant digits.
5. a) 3 b) 4 c) 3 d) 4
6. a) .1% b) .01% c) .14% d) .014%
7. The more significant digits, the smaller the per cent of error.
8. 7,812 in has the greatest accuracy and .2 in has the least accuracy.

Exercises p. 13

1. 3/4 inch
2. 7/8 inch
3. .055 inch
4. .15 inch
5. .0510 inch
6. 7/32 inch

Exercise p. 15

1.



Largest Area $= 4\frac{13}{16}$ sq. in.
Smallest Area $= 2\frac{13}{16}$ sq. in.
difference $= 2$ sq. in.
Area $= 3\frac{3}{4}$ sq. in.
or 4 sq. in.

100

Exercises pp. 17-18

1. The product would be 5 11/16 - 1/8(3 1/4 + 1 3/4) + 1/8 x 1/8
Thus we see that the error in the number of square units [illegible] the computed area is the same (except for sign) as in [illegible] previous example. The minus sign simply means the [illegible]ea might be smaller by this amount.
[illegible]48 sq. in.
[illegible] may differ by as much as 9.99 square inches
[illegible] + d)
5. [illegible] c + d)

101

UNIT X

Sample Test Questions

PART I. TRUE - FALSE.

T 1. Counting separate objects is considered to be an exact measurement.

T 2. The smaller the unit, the more precise is the measurement.

F 3. The greatest possible error would be one-sixteenth inches if the length of a line is measured to the nearest $\frac{1}{4}$ inch.

T 4. The smaller the per cent of error, the greater is the accuracy of the measurement.

F 5. If a measurement of a line is stated to be 10.0 inches, it implies that the line was measured to the nearest inch.

F 6. The more precise the measurement, the greater is the possible error.

T 7. A measurement of 300 miles has the same greatest possible error as a measurement of 700 miles.

T 8. There are four significant digits in the measurement 7,003 miles.

T 9. The term "greatest possible error" of a measurement does not refer to a mistake made in the measurement.

F 10. The greatest possible error of the sum of several approximate measurements is the same as the greatest possible error of the least precise measurement.

PART II. MULTIPLE CHOICE.

1. The most precise measurement is:

   A. $26\frac{1}{2}$ inches

   *B. 26.0 inches

   C. 260 inches

D. $26\frac{1}{4}$ in.

2. The number with the greatest accuracy is the one with the least:

A. precision.

B. possible error.

*C. per cent of error.

D. number of significant digits.

3. The greatest possible error in the sum of 45.5 in., 36.05 in., 5.1 in. is:

*A. .105 in.

B. .05 in.

C. .055 in.

D. .005 in.

PART III. COMPLETION.

1. The measurement of a line segment was stated to be $1\frac{1}{8}$ inches. The measurement might be stated as $1\frac{1}{8} \pm$ ($\frac{1}{16}$).

2. The greatest possible error in a measurement is always ($\frac{1}{2}$) the unit used.

3. A measurement of $34\frac{1}{4}$ inches has the same precision as a measurement of 5 0/ (4).

PART IV. MISCELLANEOUS.

1. Measure the length of the line segment AD to the nearest eighth of an inch. A____________D ($1\frac{4}{8}$)

2. Suppose you have measured the length and width of a rectangle each to the nearest tenth of an inch. Explain how you would find the approximate greatest possible error in the area.

($\frac{1}{20}$[length + width])

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3. Compute the per cent of error in the measurement 25 inches. How would this compare with the per cent of error in the measurement 25 miles? 1. $\left(\frac{.5}{25} = .02 = 2\text{ per cent}\right)$

2. (Same)